Networks, Information Acquisition, and Financial Markets

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Abstract

We explore a competitive financial market with non-price networks and costly information in which investors can learn from both market prices and each other. Learning from others involves direct observation of a source network and a spatial component that allows an investor to 'build' on the work of others. Price informativeness and return volatility depend on a new object we term the 'information mesh,' which incorporates total information and network interconnectedness. The magnitude of the spatial component can significantly influence the information mesh. Welfare analysis reveals significant inefficiencies whose direction depends on the initial quantity of private information.

Keywords— Information Acquisition; Networks; Spatial Economics

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1 Introduction

Over the past couple of decades, widespread use of the internet has significantly altered learning dynamics and the transmission of knowledge. In the case of the financial industry, this has led to a relative surge of non-price mechanisms of information transmission as simple and efficient alternative channels for learning.

Non-price mechanisms of information transmission include financial media services such as Bloomberg or Reuters and external consulting services such as Deloitte or McKinsey & Company, which offer rich databases used in forecasting returns. Perhaps most relevant, though, are both direct or indirect communications among investors themselves via e-mail, social networks, or slack channels. Indeed, the importance of direct communication of this sort in its more primordial forms is undoubtedly among the reasons why financial institutions have historically tended to agglomerate in particular metropolitan areas, such as New York City or London.

In this paper, we explore the consequences of exogenous non-price investor networks for market efficiency and welfare in a financial environment in the tradition of Verrecchia (1982). We consider a model of a financial market with costly information acquisition in which investors can learn about the payoff of a risky asset from prices, their own idiosyncratic signals, and the signals of others in their respective networks of sources. Learning from others affects information acquisition in two ways. First, an investor can directly learn by freely observing signals produced by others in his source network. Second, an investor can 'build' on the work of other investors by using it in the construction of his own signal.

This latter effect gets at the idea that an investor learns more than just the realization of others' signals. He also learns 'where' they have searched and thus use his energy to 'search elsewhere.' Consider, for instance, a potential investment in a Venezuelan bond. One investor in this market (A) may work to produce an accurate forecast of oil prices, which will significantly impact default risk for this particular economy. Another investor (B) who gets to freely observe the fruits of A's labor is not likely to replicate A's work by producing another set of oil price forecasts. Since he is already privy to some information in that vein, he may focus his energy instead on better understanding the political situation in the country and what it may mean for potential repayment. A third investor (C) observing either or both, may choose instead to learn more about the potential for interest rate increases in the developed world. And so on.

We model this phenomenon, which we identify as a *spatial component* to the information and payoff structure, tractably by assuming that an investor can[1] pay for a signal whose noise is *negatively* correlated with that of a source in his network. Intuitively, this implies that an investor can construct a signal that will tend to reveal information not contained in his source network. The strength of this spatial component can be intuitively expressed in terms the correlation of the noise shocks across the signals, $\iota \leq 0$. When $\iota = 0$ this effect is entirely absent, i.e., investors can still observe the signals of others in their network, but their learning efforts are independent.

The model provides several interesting insights. First, we find that the equilibrium is unique, despite the presence of significant network externalities and non-convexities in the underlying decision problems. Thus, the model offers unique predictions for the relevant comparative statics, such as how market efficiency responds to an increase in network connectedness.

Second, the model introduces a distinction between total information and a new metric we call the 'information mesh.' The former aggregates the information collection efforts undertaken by investors. The latter adjusts this figure to account for the interconnectedness of the investment network, which includes the spatial component, and is fundamentally derived from a typical investor's posterior beliefs. Two markets can coincide in total information but differ in their information meshes. The information mesh will be thicker, i.e., have a higher value, in an economy in which the (same set of) idiosyncratic signals are more widely distributed throughout the investor network.

The distinction, not present in a standard Verrecchia (1982) model without networks, is highly relevant. The information mesh is the more meaningful metric of the two, as it governs price informativeness (or *market efficiency*, as it is sometimes called, e.g., Han and Yang [2013]). Moreover, it is often the case that total information and the information mesh move in opposite directions in response to key fundamentals. For instance, an expansion of an investors' circle of sources can often reduce total information (as a result of free-riding) but nevertheless thicken the information mesh. Thus, market prices can become more informative even as total information decreases.

Third, we find that the information mesh lowers return volatility. This is because a thicker information mesh means greater market efficiency, which implies that prices more closely track payoffs, which reduces dispersion in returns. It is also the case that the information mesh thickens the portfolio correlation across

[1]Importantly, investors are also given the choice not to exploit this spatial component. It is a result, however, that they always will.

investors. This is a natural result of their information sets more closely resembling one another.

Fourth, we find that an increase in the network interconnectedness will increase market efficiency as long as information costs are high enough. This finding contrasts with that of Han and Yang (2013), who find in a related model in the vein of Grossman and Stiglitz (1980) that network growth generates enough free-riding to always reduce market efficiency. We also find free-riding to be a strong force; indeed, free-riding can often reduce *total information* in the economy. However, the *information mesh* will grow despite this as the network expands because the free-riding will not be large enough to offset the increased dispersion of information more widely throughout the economy's networks.

Fifth, the model reveals that the spatial component of information acquisition is tantamount to a reduction in the total and marginal cost of information acquisition. This implies that, ceteris paribus, greater spatial dynamics in the underlying information structure will imply larger responses of equilibrium precision to changes in fundamentals.

For instance, the magnitude of free-riding behavior is influenced substantially by the size of the spatial component of information acquisition. The bigger is the spatial component, the stronger is the effect of free-riding. The intuition is straightforward: A reduction in information acquisition efforts by investor i that results from free-riding behavior also raises marginal costs for other investors who may be building on the work done by i, which works to further reduce information acquisition efforts across the investor network.

Finally, in a welfare exercise we find that the equilibrium typically 'over-reacts' relative to the efficient allocation of information acquisition. By this we mean the following: The economy begins with some amount of private information for which investors need not exert any effort. Investors can choose to pay for additional costly signals that are then subject to the network dynamics described before. In our model, over reaction implies that when there is little initial information in the economy, the competitive equilibrium results in inefficiently low information acquisition. When there is a larger amount of initial information, though, the competitive equilibrium results in inefficiently high information acquisition.

The fundamental reason for both is that investors do not internalize the impact that their information acquisition efforts will have on market prices. In the low-information case, investors are exposed to a significant degree of consumption volatility at the end of the game and they do not want to increase it by introducing more noisy signals. The planner, though, realizes that additional information acquisition efforts will reduce return volatility, which ultimately benefits the investors.

In the high-information case, investors are exposed to very little consumption volatility at the end of the game and thus feel comfortable adding noisy signals into the mix. What the planner realizes, though, is that too much information acquisition will lower the average return, which is a first-order effect harmful to all investors but internalized by none of them. This effect is made especially harmful by the presence of the spatial component, which kicks inefficient over-acquisition into overdrive.

1.1 Related Literature and Contribution

This paper contributes to the literature on informational efficiency in financial markets spawned by Grossman (1976) and Grossman and Stiglitz (1980), who posit an environment in which investors can learn from prices but not from each other and who assume that some unlearnable uncertainty exists to preclude prices being fully revealing. Just a few examples in this literature include Verrecchia (1982), Admati (1985), Peress (2004), Dow and Gorton (2006), Van Nieuwerburgh and Veldkamp (2009, 2010), Mackowiak and Wiederholt (2009), Banerjee (2011), Valchev (2017), and Pavan et al. (2022). Colla and Mele (2010) and Ozsoylev and Walden (2011) explore similar models with information networks that are taken as exogenous. Our study extends this literature by incorporating networks into a model of endogenous and costly information production and by generating new predictions linking social communication to market efficiency and volatility.

The most related papers to ours are Halim et al. (2019) and Han and Yang (2013), who also explore financial markets with costly information and social networks. The former is an empirical study using laboratory experimental data, and the latter is the only other theoretical paper to our knowledge to study the effect of social communication on financial market outcomes when information is endogenously acquired at a cost.

There are two key differences between our work and Han and Yang (2013). The first is how information acquisition is treated. Han and Yang (2013) follow the framework of Grossman and Stiglitz (1980), in which becoming informed is a binary decision and an indifference condition between being informed and uninformed closes the model by allowing for the fraction of informed investors to adjust to clear markets.

In this framework, they show that free-riding must be a powerful force in equilibrium. This is because the expected information received from others will always be increasing as the network expands, which implies that the information received from the price, i.e., market efficiency, must be decreasing in order for this key indifference condition to hold.

We instead build on the framework of Verrecchia (1982), in which all investors are equally informed in equilibrium and information acquisition instead happens on the intensive margin. As such, the indifference condition that demands a powerful free-riding effect is not present. Free-riding continues to play a prominent role, but it need not be, and indeed is not, so over-powering as to overturn the market efficiency benefits of information sharing among the investors.

The second major difference is the presence of the spatial component, which is present in our model but is absent in Han and Yang (2013). The spatial component significantly influences learning incentives, market dynamics, and welfare.

Our results can shed partial light on some empirical experimental results by Halim et al. (2019) that seem to contrast with Han and Yang (2013). For instance, Halim et al. (2019) find that market liquidity[2] tends to rise with network interconnectedness, which accords with the prediction of our model, but not that of Han and Yang (2013).

Given that a core driver of our results is related to investors' ability to observe and mimic other investors in a social network, our paper is also related to a literature on investor herding and mimicking behavior, e.g., Hong and Stein (1999), Chari and Kehoe (2003, 2004), Veldkamp (2006a), and Gu (2011) among others.

The spatial component in our model generates a complementarity in information-acquisition activity. Many other works have argued that such information complementarities emerge in alternative settings. García and Strobl (2010) show that the marginal value of information can increase in the number of agents who acquire it, depending on how an investor's marginal utility of consumption is related to other investors' consumption. Other sources of the complementarity include short-term trades (Froot et al. [1992]; Chamley [2007]), fixed costs in information production (Veldkamp [2006b]), correlation of noise in

[2]'Market liquidity' here is interpreted as the magnitude of the pricing coefficient on the noise term. See Han and Yang (2013) for further discussion as to why.

supply and fundamentals (Barlevy and Veronesi [2007]), the presence of an additional dimension of supply information (Ganguli and Yang [2009]), feedbacks between financial markets and the value of traded securities (Goldstein et al. [2013]), or investors being ambiguity averse (Mele and Sangiorgi [2015]).

The rest of the paper is organized as follows. Section 2 describes the main features of the model and defines the equilibrium notion. Section 3 explores the key properties of the investment stage of the game. Section 4 explores the key properties of the information acquisition stage of the game. Section 5 concludes.

2 Model Description

2.1 Market Structure

We consider a financial market in the tradition of Verrecchia (1982) in which investors attempt to infer an asset's payoff by direct means, i.e., information acquisition, as well as market prices. Some aggregate unobservable noise, in this case a supply shock, obscures perfect revelation by prices. We add into this structure a social network amongst investors that allows for the transmission of some of their information via non-price mechanisms.

There are two assets traded in the market. One is a risky asset that pays a stochastic dividend, $\theta \sim \mathcal{N}\left(\bar{\theta}, \frac{1}{\kappa}\right)$ and the other yields a gross, risk-free return $R > 0$. The risk-free asset price is normalized to unity. The risky asset supply is random and distributed, $a \sim \mathcal{N}\left(\bar{a}, \frac{1}{\beta}\right)$.

On the other side of the market there is a unit continuum of investors, each denoted by $i \in [0, 1]$. These investors are rational and update their beliefs according to Bayes' rule. Following Grossman and Stiglitz (1980), they receive private signals about the underlying fundamental, θ, but can infer nothing about the risky asset supply a except through the price. They make bids contingent on both realized prices and their private information to maximize their utility post-private-signal realization. Each has an initial stock of risk-free assets, w_0.

The price of the risky asset adjusts to clear markets and can depend only on these aggregate fundamentals. It is given by a function $q(a, \theta)$. Following Grossman and Stiglitz (1980), investors observe the price and can use it to infer information, but they directly observe neither θ nor a.

2.2 Information Structure

Investors observe a countable number of private signals of the dividend, θ. Each investor, i, receives one 'non-mimickable' private signal, ξ_i, given

$$\xi_i = \theta + \nu_i$$

where $\nu_i \sim \mathcal{N}\left(0, \frac{1}{\chi}\right)$, i.e., χ is the *precision* of the non-mimickable private signal, which is taken to be exogenous. These signals follow the tradition of Verrecchia (1982) insofar as no other agent $j \neq i$ is privy to the realization of ξ_i and thus cannot condition bids on it.[3]

In addition to a non-mimickable private signal, each investor can construct a *mimickable* private signal,

$$z_i = \theta + \epsilon_i$$

where $\epsilon_i \sim \mathcal{N}\left(0, \frac{1}{\eta_i}\right)$ and $\eta_i \geq 0$ is the precision of the signal. Importantly, the precision of this signal is *chosen by the investor* in a way we'll describe shortly.

The mimickability of z_i enters in two ways. First, investor i gets to observe the realizations (and precisions) of the mimickable signals, z_j, of $M - 1 \geq 1$ other investors for some integer M. We assume that investor i can observe (and condition bids on) his own signal plus those of investors in his network which we will refer to as his *sources* and index by $s_i(2)$, $s_i(3)$, $\ldots$, $s_i(M)$. All of his sources will also be indexed in $[0, 1]$ as follows. We define a constant scaling factor $\zeta > 1$ to be such that $\zeta \in \mathbb{R}/\mathbb{Q}$, i.e., ζ is an irrational number. We then define $s_i(k) = i/(\zeta k)$ when $k > 1$. Because each investor constructs his own signal as well and is thus among his sources, we set $s_i(1) = i$.

This particular network structure is designed to eliminate strategic incentives. The subset of i's mimickers will always be countable and so investor i could never hope to influence market prices by swaying the mimicry of others. Further, while every investor $j \leq 1/\zeta$ will be a member of some other investor's source network, due to the irrationality of ζ, no two source networks will be overlapping.[4]

The second way mimickability plays a role is through a spatial component in the information structure

[3] χ controls the amount of initial information in the economy before agents contemplate adding more to it endogenously. It turns out to be a critical parameter for welfare analysis.

[4] Further, when j is a member of i's network, j's source network will never intersect with the rest of i's network. This also follows from the irrationality of ζ.

itself. We model this as follows: An investor i may select at most one signal, j, from his source network to 'build on.' We will call this source $s_{b,i}$. He does so via a technology that sets the innovation in his mimickable signal (ϵ_i) to be correlated with the innovation in j's mimickable signal, (ϵ_j). Thus, the two innovations taken individually are marginally as described above but, taken together, are jointly normal with a correlation coefficient given by a fixed parameter $\iota \in (\underline{\iota}, 0]$.

We will show this formally in the analysis, but intuitively the negativity of ι captures the idea that, after observing the realization of j's signal, i can choose to search 'in a different direction,' and learn information that j's signal does not tend to reveal.[5] The strength of this effect is given by the distance of ι from zero. When $\iota = 0$, this effect is absent and all information-gathering is orthogonal to what is collected from the source network. As ι falls away from zero, the investor will be able to save on information costs by building on prior knowledge from the source network in this way.

The construction of mimickable signals is subject to a convex information cost, $C(\eta_i)$ that satisfies (1) $C(0) = 0$, (2) $C'(\eta) \geq 0$, (3) $C''(\eta) \geq 0$, and (4) $C'(0) = 0$. There is also a constant per-unit disutility of the information cost given by $\lambda > 0$.

In addition to these mimickable (semi-private) signals, investors can also infer information from the price itself, which in equilibrium will serve as a noisy aggregate signal of the dividend as well.

It is most convenient to express the investors' problem working backward in time. We follow Grossman and Stiglitz (1980) and many others in their tradition and assume constant absolute risk aversion in the utility function with absolute risk-aversion given by $\alpha > 0$. This implies that the investment problem for investor i, conditional on a given information set, is given by

$$U_i\left(\xi_i, \{z_{s_i(k)}\}_{k=1}^M, q, \{\eta_{s_i(k)}\}_{k=1}^M\right) = \max_{b_i} \ \alpha\mathbb{E}\left[-\exp\left(-\alpha\tilde{c}_i\right)|\xi_i, \{z_{s_i(k)}, \eta_{s_i(k)}\}_{k=1}^M, q\right] \tag{1}$$

$$c_i = \left[w_0 - qb_i\right]R + b_i\theta$$

The objects of the expectation are random variables denoted with a tilde and are taken with respect to the investor's information set at the time. There are no financial frictions here and investors are allowed to short the risky asset.

[5] The irrationality of ζ also ensures that an investor can never build on a signal that is already built on another signal in his source network, as it ensures that the source networks are non-overlapping.

Moving backward from the investment decision, the ex-ante utility in the information design problem is given by the payoff of a Nash game in information acquisition, wherein each investor takes the signal precisions of the other investors as given.

$$U_i\left(\{\eta_{s_i(k)}\}_{k=2}^{M}\right) = \max_{0\leq\eta_i\leq\bar{\eta}, s_{b,i}\in\{\{s_i(k)\}_{k=2}^{M},\phi\}} \mathbb{E}\left[U_i\left(\tilde{\xi}_i, \{\tilde{z}_{s_i(k)}\}_{k=1}^{M}, \tilde{q}, \{\eta_{s_i(k)}\}_{k=1}^{M}\right)\right] - \lambda C\left(\eta_i\right) \tag{2}$$

where $s_{b,i}$ is the choice of which investor in the source network to build on, the procedure for doing which is described above. Notice that the investor may also choose to not build on any source in his network, as denoted by the presence of the empty set among the choices for $s_{b,i}$. Setting $s_{b,i} = \phi$ would imply a choice to set the noise of his signal to be independent of all others in his network.

Notice that the information cost applies only to mimickable information production as the non-mimickable private information is exogenous. Also, while investors can make bids contingent on price realizations, they cannot make information acquisition decisions based on it. This is consistent with the idea of price-contingent bids as being limit orders that must be placed ahead of time.[6]

The solution to this problem is a *best-response* function. For analytic tractability, we will assume that investors in the first (information-gathering) stage are optimizing a second-order approximation of the objective function in Problem 2 and define the best-response function from this solution to be

$$\eta_i^* = G_I\left(\{\eta_{s_i(k)}\}_{k=2}^{M}\right) \tag{3}$$

2.3 Market Clearing

The price, $q(a,\theta)$, must adjust to clear markets in all possible aggregate states, i.e.,

$$a = \int_0^1 \int\int\cdots\int b_i^*\left(\xi_i, \{z_{s_i(k)}^*\}_{k=1}^{M}, q(a,\theta)|\{\eta_{s_i(k)}^*\}_{k=1}^{M},\right) f(z_{s_i(1)}|\theta)dz_{s_i(1)}\dots f(z_{s_i(M)}|\theta)dz_{s_i(M)} f(\xi_i|\theta)d\xi_i di \tag{4}$$

where b_i^* is the solution to the investment problem, i.e., Equation 1 and η_i^* is the solution to the information game, i.e., the fixed point of the best-response function implied by Equation 2.

[6]The cap on information acquisition is merely a technical condition. In practice we will set $\bar{\eta}$ high enough that it never binds in equilibrium.

2.4 Equilibrium Definition

A **Competitive Equilibrium** will be a price function, $q(a, \theta)$, investment policy functions, b_i^*, and information acquisition level, η^*, such that

1. b_i^* maximizes investor utility in Equation (1).
2. $\eta^* = G_I\left(\{\eta^*\}_{k=2}^M\right)$, i.e., the information acquisition satisfies Equation (3) and agents are homogenous at the information-gathering stage.
3. Markets clear, i.e., Equation (4) holds.
4. The pricing function is linear in its inputs.

3 Analysis: Investment and Pricing

In this section, we formalize a collection of useful and interesting results regarding the model's behavior in equilibrium. Proofs of all propositions can be found in Appendix A.

3.1 Uniqueness and Efficiency

We begin with a uniqueness result.

Proposition 1. *A unique competitive equilibrium exists.*

Existence is not terribly surprising, but uniqueness is. The network dynamics and spatial component generate strong strategic complementaries in information acquisition stage. Further, the investor's equilibrium information acquisition problem is actually not convex.[7] Despite these hurdles, though, a unique equilibrium emerges in which the allocation of investment resources and information is typically governed by marginal trade-offs.

To help build intuition, we write out the equilibrium price following Admati (1985) as follows

$$Rq(a, \theta) = A + B \times (a - \bar{a}) + C \times \theta \tag{5}$$

[7] See the proof of Proposition 1 for details regarding the non-convexity.

where A, B, and C are expressions dependent on η^*. This is useful because we can interpret the equilibrium price as a signal of the dividend whose imprecision is driven by the independent, aggregate, and mean-zero shock $\tilde{a} - \bar{a}$. Since the various slopes of the pricing function depend on η^*, the degree to which the supply shock obscures this information depends on the equilibrium acquisition of information. This can be seen by re-writing the pricing expression as

$$\frac{Rq}{C} - \frac{A}{C} = \theta + \frac{B}{C}(a - \bar{a}) \tag{6}$$

That is, when investors observe a market price, q, they can transform via a series of constants to arrive at an expression that can be interpreted as a signal of the fundamental. This signal will have a precision given by

$$\rho_\theta = \frac{C^2}{B^2}\beta$$

that we will refer to as the *market efficiency* or *price informativeness*. Such terminology is common in the literature, e.g., Ozsoylev and Walden (2011) or Peress (2010).

The first relevant feature of our model that emerges is a distinction between *total information* and the *information mesh* for the purpose of price informativeness. **Total information** refers to the integration over all private information collected in the market and is equal to $\chi + \eta^*$. The **information mesh** describes both how much information is acquired in total *and* the degree to which it is tangled and entwined across the various networks of investors. It is given by

$$I\hat{M} = \chi + \left(M - 1 + \frac{(1-\iota)^2}{1-\iota^2}\right)\eta^* \tag{7}$$

The information mesh is fundamentally derived from the non-price component of the posterior information set of a typical investor. It is driven by (1) the accuracy they observe/collect themselves, $\chi + \eta^*$, (2) the accuracy of the information observed in the rest of the network, $(M-1)\eta^*$, and (3) the effective benefit they derive from the spatial component,[8] i.e., $[(1-\iota)^2/(1-\iota^2) - 1]\,\eta^* \geq 0$, where the inequality holds true whenever $\iota \leq 0$ and will be larger the further ι is from zero. The sum of these three terms delivers the information mesh.

[8]The exact reason why investors benefit in this way will be explored in Proposition 5 and its associated discussion.

Total information and the information mesh play different roles in the model, with the latter generally being more important. This is seen first in Proposition 2.

Proposition 2. *Market efficiency, ρ_θ, is increasing in $I\hat{M}$ and not necessarily in total information.*

In a Verrecchia (1982) model without networks, market efficiency typically moves with total information. This is because the price reflects shifts in demand driven by the information acquisition undertaken by individual investors and, in a standard framework, their information is completely private.

3.2 Volatility and Portfolios

In this section, we derive a pair of results that will prove useful both for our later analysis and for future empirically minded researchers. They require the imposition of only a couple of minor technical assumptions, which we will maintain throughout.

In particular, we can link the information mesh, which is unobservable but crucial for equilibrium dynamics, to return volatility and portfolio correlations across investors, both of which are in principle observable.

We begin with a pair of additional assumptions.

Assumption 1. *The overall volatility in the economy is sufficiently large relative to risk-aversion. In particular, $\sqrt{\kappa\beta} < \alpha$*

Assumption 2. *The following technical condition holds: $\alpha\bar{a} \approx \kappa\bar{\theta}$*

These assumptions turn out to be sufficient to generate the implication that return volatility falls as the information mesh thickens. The former ensures that the variance in the price that results from the noisy supply shrinks as the information mesh thickens; the latter eliminates the impact of the constant A term in the price, which allows for the closed-form results that follow.

Proposition 3. *Return volatility is decreasing in the information mesh.*

This is intuitive. As prices become more informative, which happens as the mesh thickens, prices tend to more accurately reflect the fundamental payoff. This implies less dispersion in returns overall, as prices are higher when payoffs are higher and lower when payoffs are lower.

This does not mean that *price* volatility is necessarily lower. Proposition 3 follows fundamentally because noise in the price reduces with the information mesh. But this effect may be offset by an increase in the variance that follows from the price more closely tracking the volatile asset payoff. In fact, it may happen that the mitigation in return volatility can only occur as a result of prices becoming more volatile. But from the perspective of investor welfare, return volatility is what matters.

This finding has the additional implication that future researchers might use return volatility as a proxy for the all-important information mesh, especially in conjunction with the next result, which provides an alternative measure of the information mesh, i.e., the correlation across investor portfolios:

Proposition 4. *The correlation across random investor portfolios is increasing in the information mesh.*

By 'random investors' here, we mean two randomly drawn investors that are not in the same network, which is a probability one event when drawing a random pair, as source networks are finite but investors exist on a continuum. The result would still hold if we conditioned on investors in the same network.

Thus, if empirical researchers exploring a similar environment isolate a factor that simultaneously reduces return volatility while increasing portfolio correlations, it is likely they are honing in on the information mesh, which is a monotonic predictor of market efficiency.

4 Analysis: Information Acquisition

With the investment and pricing dynamics well-defined and understood, we are ready to proceed to the earlier, more pivotal stage: The information acquisition game that is played among investors before investment takes place.

4.1 Spatial Component

The spatial component of information acquisition generates a powerful externality that influences market efficiency and welfare. This can be seen first in the following result

Proposition 5. *Problem 2 is isomorphic to the following problem*

$$U_i\left(\{\eta_{s_i(k)}\}_{k=2}^{M}\right) = \max_{0\leq\eta_i\leq\bar{\eta}} \mathbb{E}\left[U_i\left(\tilde{\xi}_i, \{\tilde{z}_{s_i(k)}\}_{k=1}^{M}, \tilde{q}, \{\eta_{s_i(k)}\}_{k=1}^{M}\right)\right] - \lambda\hat{C}\left(\eta_i;\eta_j\right) \tag{8}$$

where

1. ϵ_i *is independent from all other innovations in the source network.*

2. $\eta_j = \max\{\eta_{s_i(2)}, \eta_{s_i(3)}, \ldots, \eta_{s_i(M)}\}$

3. *The cost function*[9] *is given by*

$$\hat{C}\left(\eta_i;\eta_j\right) = C\left(\left(\sqrt{(1-\iota^2)\eta_i} - \sqrt{\iota^2\eta_j}\right)^2\right)$$

In words, we can represent the problem with spatial searching as a problem without spatial searching but for which an information cost externality exists. We observe also that the investor always builds on the work of others; it is never optimal to not exploit this opportunity. We can further say the following about this new cost function.

Corollary 1. *Define* $\underline{\eta} = \iota^2/(1-\iota^2)\eta_j$. *The information cost function,* $\hat{C}$ *exhibits the following properties on the domain* $(\underline{\eta}, \bar{\eta}]$

1. $\hat{C}(\underline{\eta};\eta_j) = 0$.

2. $\hat{C}'(\underline{\eta};\eta_j) = 0$.

3. $\hat{C}'(\eta;\eta_j) \geq 0$ *for all* $\eta \geq \underline{\eta}$.

4. $\hat{C}''(\eta;\eta_j) \geq 0$ *for all* $\eta \geq \underline{\eta}$.

5. $\hat{C}'(\eta;\eta_j) \leq C'(\eta)$ *for all* $\eta \geq \underline{\eta}$.

We observe a few fascinating properties in this externality. First, the presence of spatial searching makes some levels of precision free. In particular, those in $[0, \underline{\eta}]$. Further, the marginal cost at $\underline{\eta}$ is zero, so the agent would always choose a bit more information than this provided there is any benefit whatsoever from doing so.[10]

Second, the cost function continues to be increasing and convex on the domain over which it applies. And third, marginal costs in the benchmark model are always lower as a result of the spatial searching.

[9] This functional form reveals that not all ι can be consistent with a competitive equilibrium. In particular, a cap of $\bar{\iota} = -1/\sqrt{2} \approx -.707$ is required to ensure effective precision choice remains positive when precisions are homogeneous across investors. This restriction implies that $1-\iota^2 \geq \iota^2$.

[10] It is worth noting that this does not eliminate corner solutions. It can be, and often is, the case that the additional consumption volatility induced by any information acquisition in the first stage makes the agent worse off.

All of these imply the following.

Corollary 2. *The information mesh thickens as ι falls away from zero.*

Thus, the spatial component generates a significant positive externality that boosts information acquisition efforts for all investors, which in turn increases the information mesh. All of the previous results are then implied, i.e., increased market efficiency, reduced return volatility, etc.

4.2 Network Comparative Statics

Having established that the information mesh is the most relevant object, it is clear that the most relevant comparative static for our purposes is how the information mesh responds to changes in network interconnectedness, i.e., M. The answer is not obvious as there are multiple forces acting on the information mesh. An increase in M mechanically increases the information mesh fixing η, but η will not generally remain fixed in the process. Free-riding forces will increase, which will work to reduce η in equilibrium. It is also the case that the average return of the risky asset as well as the implied return and consumption volatility will influence and be influenced by η as the economy's networks deepen.

Despite all of this, we can provide a simple sufficient condition that ensures that these offsetting forces are never large enough to overwhelm the gains to the information mesh that obtain when M increases information sharing across investors.

Proposition 6. *If the information cost, λ, and the initial private information, χ, are both sufficiently high, then the information mesh is increasing in M.*

These conditions are sufficient and not necessary, but the intuition behind them is as follows. If it were not for the response of the equilibrium prices to changes in η^*, the result would follow without these conditions. To see this, we present a brief intuitive argument by contradiction. In this hypothetical fixed-price case, the marginal benefit of acquiring more information would shrink with the information mesh.[11] Let us suppose, then, that the information mesh, $I\hat{M}$, shrank with M. If this were true, then the overall marginal benefit of information would *increase* with M. But the information mesh can only be shrinking with M if free-riding forces are strong enough to reduce total information (η^*).[12] Given the convexity of

[11] See the proof of Proposition 1 for an argument as to why.

[12] This is because there is a mechanical increase in the information mesh as M goes up that can only be offset if information acquisition falls.

the cost function, a reduction in η^* would imply a *strict decrease* in the marginal *cost*. In equilibrium, the marginal benefit of information must equal the marginal cost of information. Under the assumption that $I\hat{M}$ shrinks in response to M growing, the former must increase while the latter strictly decreases, which is impossible. Thus, it must be that $I\hat{M}$ increases as M grows when prices are fixed.

The conditions in Proposition 6 ensure that additional general equilibrium effects of the information mesh on the price are not strong enough to upset this basic argument. A high value of χ works to mute the derivatives of the pricing coefficients with respect to the information mesh, as it exogenously increases the information mesh and these expressions are concave in the information mesh. A high value of λ ensures that the reduction in marginal cost effect described in the previous paragraph dominates any lingering equilibrium effects from changing prices.

We can see Proposition 6 in action in the example[13] in Figure 1. The main impact of the network on investor behavior regardless of the presence of the spatial component is to induce free-riding. An increase in the network size causes total information to fall mildly.[14] This fall, however, is not so great that it causes the information mesh to attenuate, as the mesh is also increasing in the raw size of the source network. Thus, the increased interconnectedness of the network more than overcomes the free-riding. This thickens the information mesh and, by Propositions 2 and 3, increases market efficiency and reduces return volatility.

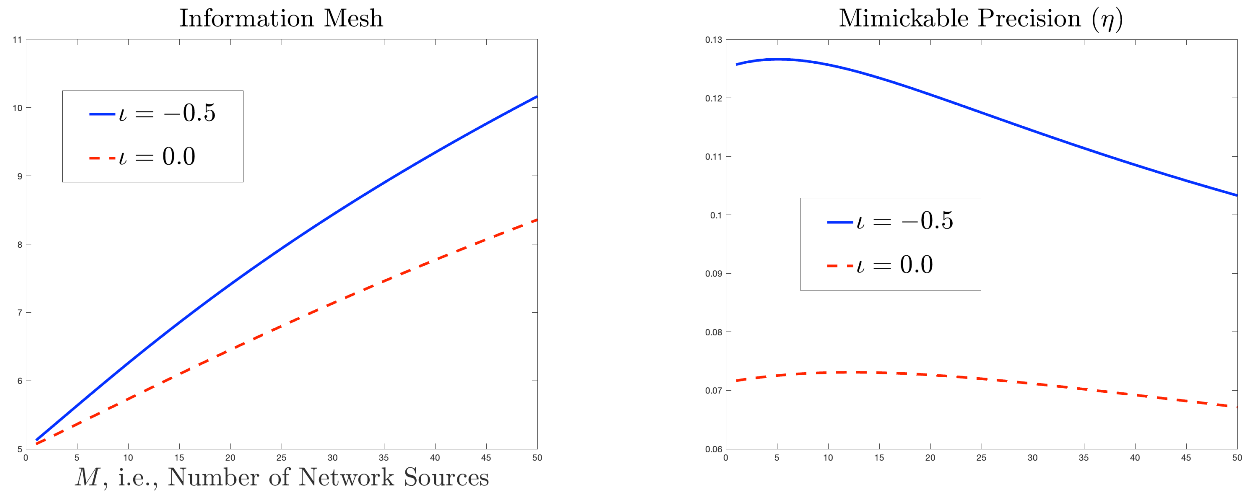
The spatial dynamics are also important here. The positive externality it generates is manifested strongly. An ι of higher magnitude implies more information collected and a thicker information mesh relative to the independent case.

ι also governs the size of the free-rider effect as M increases. This can be seen in Figure 2, which gives the percentage change in equilibrium precision as the network deepens. Universally it is the case that a more negative ι results in a faster decrease in information efforts as M increases.

The intuition is that a reduction in effort on the part of investor i as a result of free-riding becomes amplified in equilibrium by virtue of the fact that investor i's lower precision raises the marginal cost of

[13] See Appendix B.1 for the parameterization.

[14] The mild initial increase in η for low levels of N is an equilibrium effect driven by expected return and consumption dynamics that more than overcome the free-rider effect initially. It is one of those equilibrium price effects whose impact on the information mesh is rendered relatively small by the conditions of Proposition 6. It is worth noting that despite this, these price effects are nevertheless important for welfare and equilibrium efficiency and will be discussed in some detail in Section 4.3.



(a) **(b)**

Figure 1: Comparative Statics: Network Size

acquiring information for any investor that builds on his work. In equilibrium, this will imply a larger decrease in precision for all investors.

4.3 Welfare

Networks become functionally irrelevant once information sets are determined. Consequently, they have no independent impact on the second (investment) stage of the game.[15] As such, we focus our analysis on the preceding information acquisition stage, where the presence of networks significantly impacts equilibrium dynamics. In the analysis that follows, we will compare the equilibrium values of information acquisition to their efficient levels conditional on both going on to generate equilibrium prices and demand functions in the investment stage that follows.

During the information acquisition stage there are three relevant[16] externalities that the social planner accounts for that private investors in a competitive equilibrium do not.

First, investor i does not internalize the fact that his information acquisition reduces the marginal cost of information acquisition for other investors by providing them a foundation on which to build. By reducing marginal costs in this way, this externality generates *more* information acquisition ceteris paribus.

[15] For a treatment of the efficiency properties of the investment stage of a similar model (albeit without networks), we refer the reader to Pavan et al. (2022).

[16] There are more externalities than these, but these are the ones necessary to understand the intuition of the welfare analysis.

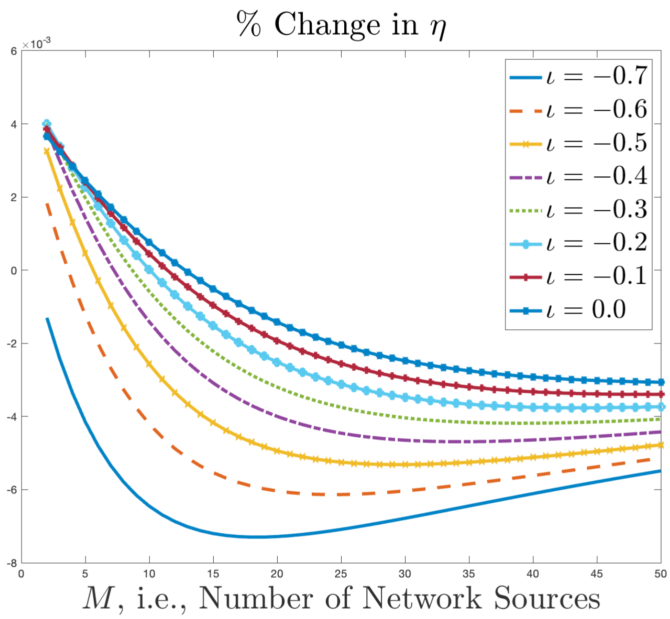


Figure 2: Free-Riding Across ι

Second, there is a pecuniary externality. Investors do not internalize that their choice of η_i will impact both the average return and, through it, the implied volatility of investment in response to the fundamental.

Third, there is a learning externality. Investors do not internalize that their choice of η_i will influence how informative the price will be in equilibrium, as they take the pricing structure (and hence its information content) as given.

The social planner's problem addresses these externalities by amending Problem 2 as follows: (1) The social planner maximizes the same objective by selecting all $\{\eta_i\}_{i\in[0,1]}$ simultaneously, as opposed to just one at a time, and (2) the social planner treats the pricing terms, i.e., A, B, and C, as functions of $\{\eta_i\}_{i\in[0,1]}$ as opposed to taking them as fixed.

As a result of these myriad externalities and in contrast to the competitive equilibrium, the social planner's problem is highly non-convex and thus difficult to characterize analytically. However, it is quite easy to construct numerical examples[17] and to get a sense of how these competing externalities affect the solution.

It is easiest to understand the difference between the efficient allocation and the equilibrium one by

[17] We use global solution methods when solving the planner's problem numerically to account for the non-convexities driven by the various externalities. Parameter choices are found in Appendix B.1.

exploring the role of the C-term in the pricing function, which pre-multiplies the asset payoff itself. C serves two important functions. First, it is key in determining how much of the variance in the price is due to payoff fluctuations as opposed to noise. Second, it governs the average expected return, as the average gross return is decreasing in C. Of equal importance is the fact that a larger average gross return implies larger swings in optimal investment and thus consumption volatility in the investment stage.

It is also the case that C is directly related to the information mesh. In particular, as the information mesh thickens, C increases.[18] This lowers the average return as well as investment and consumption volatility.

We find that the equilibrium tends to 'overreact' to the informativeness of the market. When χ is low, which exogenously reduces the information mesh, we get a low value of C. This implies high average returns, but also a great deal of investment and consumption volatility. Given how volatile consumption already is, private investors do not wish to invest any more in private information, as this will generate even more volatility in the second stage.[19] As such, they set $\eta^* = 0$, which is a corner solution in which no mimickable signals are collected and total information, as well as the information mesh, is just given by χ.

Investors do not internalize the fact that additional information acquisition will, in equilibrium, reduce their consumption volatility. This happens by means of two channels. First, prices become more informative as market efficiency will rise. This reduces the effective uncertainty to which the investors are exposed in the investment stage. Second, the average return itself decreases and, with it, the magnitude of the swings in the investment positions. The planner realizes these effects, however, and would account for them by forcing investors to acquire some information. This effect can be observed in Figure 3.

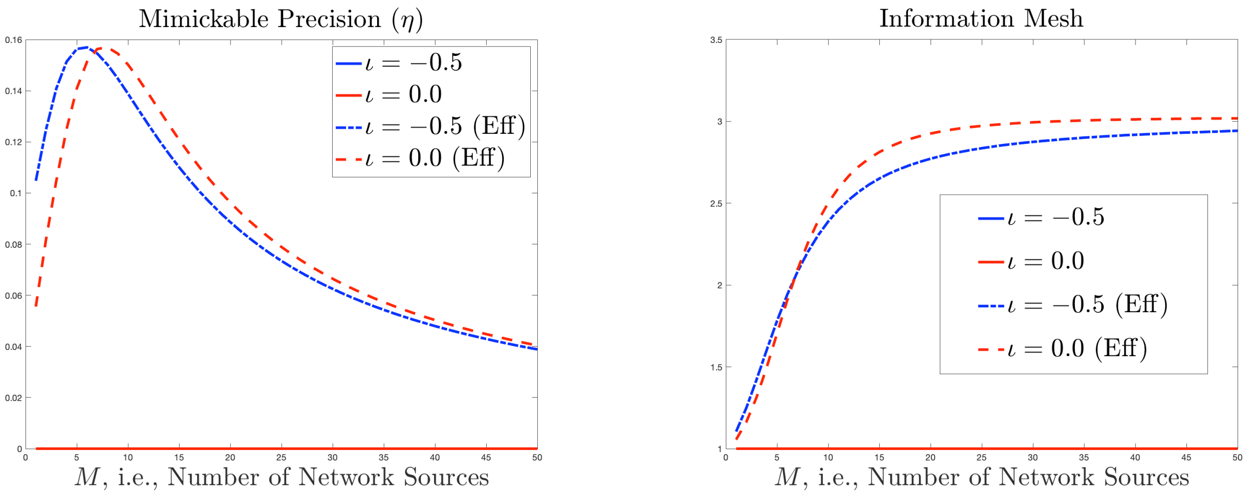
The opposite occurs when χ is high, however. In this case, prices are exogenously very informative and returns are expected to be lower. This implies that there is relatively little investment and consumption volatility. Investors can afford to shoulder a bit more of the consumption risk that comes from more accurate signals. Thus, they invest even more in private information.

In equilibrium, though, this makes expected returns too low, which reduces the welfare of all investors. The planner can do better by reducing information acquisition all the way to the corner of zero.[20] This

[18] See Appendix A for an argument.

[19] The signals generated by the private information will reduce uncertainty *ex post*, but *ex ante* it introduces additional shocks into the investment policy.

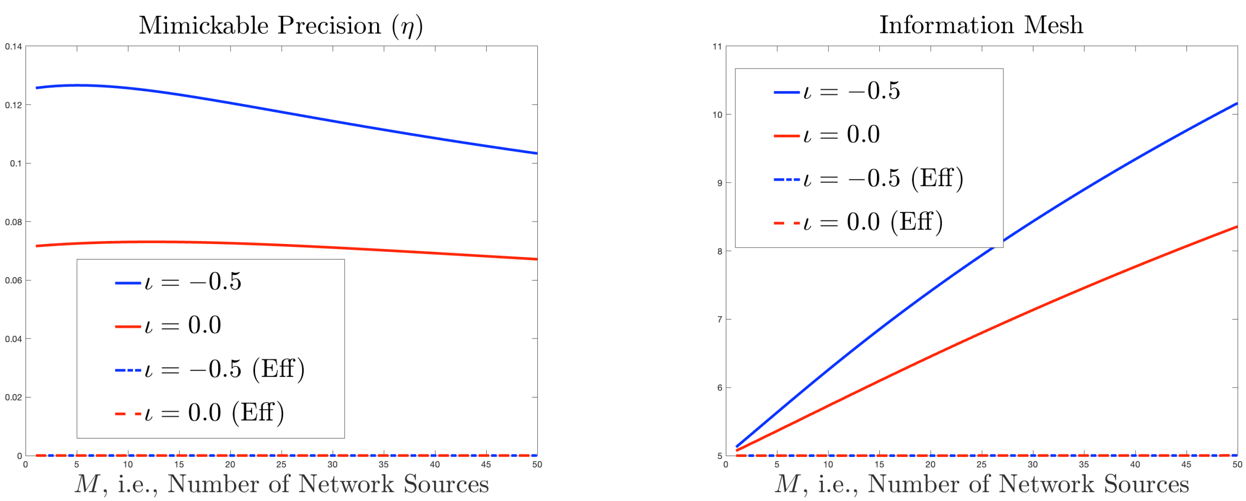
[20] In all cases we examined, we never encountered a solution in which the equilibrium and efficient allocations were jointly were either both



(a) **(b)**

Figure 3: Efficiency Results: $\chi = 1.0$

effect can be observed in Figure 4.



(a) **(b)**

Figure 4: Efficiency Results: $\chi = 5.0$

Importantly, the *magnitude* of this over-investment is determined in large part by the network and the spatial component. Figure 5 reveals that the gap between the efficient allocation and the equilibrium

corners or both interior. In all cases we could examine, we found inefficiently low information levels for low χ that eventually became inefficiently high as χ increased. This switch would only take place when the equilibrium would leave a corner solution and, at the same time, the planner's solution would degenerate into one.

widens acutely as the network grows. It also reveals that a substantive part of this widening is due to the spatial component making information too cheap, as can be seen in the tight linkage between the information mesh and the welfare cost.

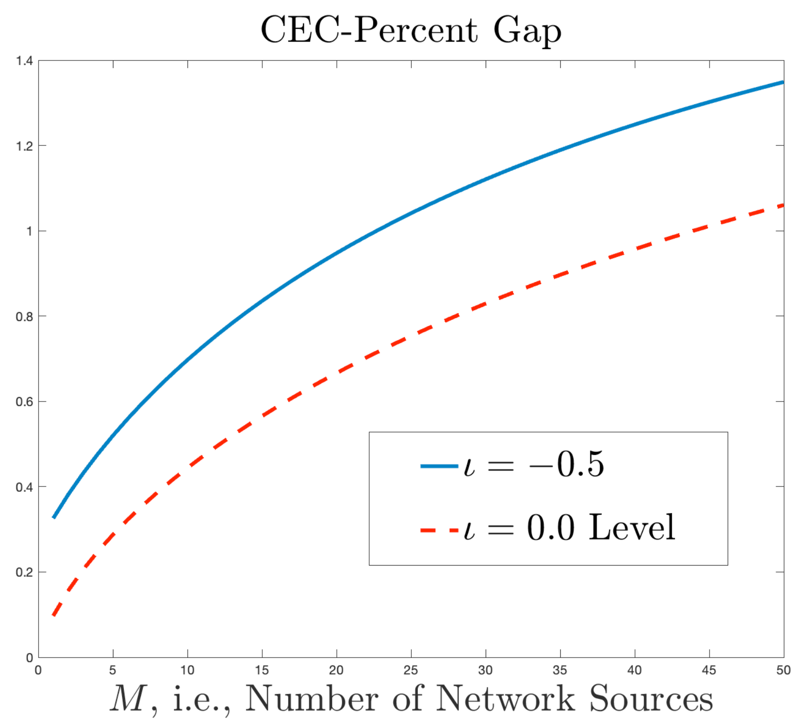


Figure 5: Welfare Results: $\chi = 5.0$

These results suggest that policy implementation designed to rectify market inefficiencies would be difficult in such a market. Whether there is significant over or under-investment in information largely depends on the aggregate quantity of hidden information, which is a notoriously difficult thing to measure empirically. Further, the inefficiencies arise as a product of several externalities, and not just one. Thus, a policy-maker would likely require several interactive policy instruments.

5 Conclusion

In this paper, we explored a competitive financial market with endogenous information acquisition and social networks. We demonstrated that the information mesh, not total information, is the crucial information variable that governs market efficiency. Further, how the information mesh responds to key parameter changes depends critically on the magnitude of the information production externality.

A natural next step is for empirical researchers to take up the task of exploring some of the model's key predictions. It is easy to show that return volatility or portfolio correlations could be used as a proxy for the information mesh in such exercises.

A Proofs

We will not prove the results in the chronological order presented in the paper, but rather in the most natural order in which to solve the model. Thus, we will begin with the isomorphism result for the spatial component and use it in the solution of the rest of the model.

A.1 Proof of Proposition 5

To establish this, we first analyze investor i's signal.

$$z_i = \theta + \frac{1}{\sqrt{\eta_i}}\epsilon_i$$

where ϵ_i is a standard normal. Since ϵ_i is correlated with ϵ_j, though, we can alternatively express this as

$$z_i = \theta + \frac{1}{\sqrt{\eta_i}}\underbrace{\left[\iota\epsilon_j + \sqrt{1-\iota^2}\hat{\epsilon}_i\right]}_{\epsilon_i}$$

where $\hat{\epsilon}_i$ is a standard normal that is uncorrelated to all other shocks. We can substitute in $\epsilon_j = (z_j - \theta)\sqrt{\eta_j}$ and manipulate it to derive the effective information content of the correlated signal, i.e.,

$$\frac{z_i - \iota z_j\sqrt{\frac{\eta_j}{\eta_i}}}{1-\iota\sqrt{\frac{\eta_j}{\eta_i}}} = \theta + \underbrace{\frac{\sqrt{\frac{1-\iota^2}{\eta_i}}}{1-\iota\sqrt{\frac{\eta_j}{\eta_i}}}}_{1/\sqrt{\hat{\eta}_i}}\hat{\epsilon}_i$$

We observe that, because of the correlation, the agent takes the (transformed) difference between his signal and the one on which he built. This supplants his original signal with another with precision $\hat{\eta}_i$. It's also clear that when ι goes to zero, this expression boils down to investor i's original signal, as one would expect.

We can manipulate the expression for $\hat{\eta}_i$ as follows

$$\sqrt{\hat{\eta}_i} = \frac{\sqrt{\eta_i} - \iota\sqrt{\eta_j}}{\sqrt{1-\iota^2}}$$

which implies that $\hat{\eta}_i \geq \eta_i$ provided $\iota \leq 0$, i.e., the agent effectively 'gets' more precision than he pays for by building off of j's signal. This is driven by two components. There is an uncertainty reduction component in the denominator that increases $\hat{\eta}_i$ regardless of the sign of ι, and there is a direction component in the numerator that will only increase $\hat{\eta}_i$ if $\iota \leq 0$. If the correlation was positive, this would work instead to reduce the new posterior precision, as the new signal will tend to reveal information that has already been revealed.

If ι was positive, it would be ambiguous which effect would dominate, i.e., the uncertainty reduction or the direction effect. Under our assumption of a negative ι, though, these two forces work together to unambiguously increase posterior precision. For this reason, investors will always choose to build on the work of others when given the choice.

It is also easy to manipulate this further to derive an expression of η_i in terms of $\hat{\eta}_i$

$$\begin{aligned} \eta_i &= \left(\sqrt{(1-\iota^2)\hat{\eta}_i} + \iota\sqrt{\eta_j}\right)^2 \\ \implies \eta_i &= \left(\sqrt{(1-\iota^2)\hat{\eta}_i} - \sqrt{\iota^2\eta_j}\right)^2 \end{aligned} \tag{A.1}$$

where the last equality follows from the fact that $\iota \leq 0$.

The proposition follows from the fact that an agent paying for an iid precision $\hat{\eta}_i$ is achieved by paying for a correlated signal η_i which will always be less that $\hat{\eta}_i$. The relationship between the two is given by Equation A.1, which is where the new cost structure comes from. It is further observed in this last expression that greater values of η_j will lead to lower effective costs, so the agent will always build off of the most precise signal in his information set.

It is fairly straightforward to derive the properties outlined in Corollary 1. First, we observe that the cost will only be defined when $\sqrt{(1-\iota^2)\hat{\eta}_i} \geq \sqrt{\iota^2\eta_j}$, which establishes the lower bound $\underline{\eta}$ described in the proposition. It is also worth noting that when $\hat{C}(\underline{\eta};\eta_j) = 0$, so choosing this lower bound is costless.

It is also obvious that η_i is increasing in $\hat{\eta}_i$, and thus $\hat{C}$ is an increasing function whenever $\hat{\eta}_i \geq \underline{\eta}$. Nevertheless, it will be worthwhile to take the first-order condition for three reasons: First, to inspect its value at $\underline{\eta}$; second, to inspect how the marginal cost is impacted by the efforts of investor j; and third, to lay the groundwork for a second-order condition to verify convexity. The first-order condition is

$$\frac{\partial \hat{C}}{\partial \hat{\eta}_i} = C'(\eta_i) \times 2 \times \left(\sqrt{(1-\iota^2)\hat{\eta}_i} - \sqrt{\iota^2\eta_j}\right) \times \frac{1}{2}\sqrt{1-\iota^2} \times \hat{\eta}_i^{-1/2}$$

This expression is always positive and, further, equals zero at $\hat{\eta}_i = \underline{\eta}$. It's also straightforward to see that (because η_i is decreasing in η_j)

$$\frac{\partial^2 \hat{C}}{\partial \hat{\eta}_i \partial \eta_j} \leq 0$$

i.e., the marginal cost of i's effective precision is falling in the efforts of j.

Finally, we derive the second-order condition as follows:

$$\frac{\partial^2 \hat{C}}{\partial \hat{\eta}_i^2} = C''(\eta_i) \times \left((1-\iota^2) - \sqrt{\frac{\iota^2(1-\iota^2)\eta_j}{\hat{\eta}_i}} \right)^2 + C'(\eta_i) \times \left(\sqrt{\iota^2(1-\iota^2)\eta_j} \right) \times \frac{1}{2} \times \hat{\eta}_i^{-3/2} \geq 0$$

which establishes convexity.

To see Corollary 2, we observe that, due to the convexity of $\hat{C}$, the drop in the marginal cost associated with ι falling away from zero will increase the optimally chosen level of $\hat{\eta}_i$ (and thus η_i). This is verified by the fact that the marginal benefit of precision will always be weakly decreasing, a result that we establish in the proof of Proposition 1.

A.2 Proof of Propositions 1 and 2

Observe that under CARA preferences, the FONC implies that the optimal investment policy has the form

$$b_i^* = \frac{1}{\alpha \text{Var}_i(\tilde{\theta})} \times \left[\mathbb{E}_i \left[\tilde{\theta} \right] - qR \right]$$

where the subscript i denotes an expectation or variance taken with respect to the investor's information set.

To solve for the equilibrium, we follow Admati (1985) and take a guess-and-verify approach. In particular, we conjecture a linear pricing rule of the form

$$Rq(a, \theta) = A + B \times (a - \bar{a}) + C \times \theta$$

for real constants A, B, and C. The first convenient thing about this is that it allows us to treat the price as a signal of θ with the noise coming from the supply shock/noise traders,

$$\frac{Rq}{C} - \frac{A}{C} = \theta + \frac{B}{C}(a - \bar{a})$$

The precision of this signal will be the inverse of the variance of the mean-zero noise, i.e., $\frac{C^2}{B^2}\beta$.

The Gaussian structure of the signals (including the aggregate price) allows for a convenient description of the conditional variances, also described by Veldkamp (2011) among others, i.e.,

$$\theta|\xi_i, \{z_{s_i(k)}\}_{k=1}^M, q \sim$$
$$\mathcal{N}\left(\frac{\chi\xi_i + \sum_{k=2}^M \eta_{s_i(k)} z_{s_i(k)} + \hat{\eta}_i \left(\frac{z_i - \iota z_j\sqrt{\frac{\eta_j}{\eta_i}}}{1-\iota\sqrt{\frac{\eta_j}{\eta_i}}}\right) + \frac{C^2}{B^2}\beta\left(\frac{Rq}{C} - \frac{A}{C}\right) + \kappa\bar{\theta}}{\chi + \sum_{k=2}^M \eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa}, \frac{1}{\chi + \sum_{k=2}^M \eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa}\right)$$

where we invoke the notation defined in the proof of Proposition 5 and their associated results, namely the definition of $\hat{\eta}_i$ (which can be constructed from η_i, η_j, and ι) and the fact that the investor constructs a more precise signal by comparing it to z_j, which we take to be the signal of the source with the most precision in the network, i.e., the highest η_j.

Plugging these expressions into the policy function delivers a policy rule[21] in terms of fundamental parameters.

$$b_i^* = \frac{\chi + \sum_{k=2}^M \eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa}{\alpha} \times \left[\frac{\chi\xi_i + \sum_{k=2}^M \eta_{s_i(k)} z_{s_i(k)} + \hat{\eta}_i \left(\frac{z_i - \iota z_j\sqrt{\frac{\eta_j}{\eta_i}}}{1-\iota\sqrt{\frac{\eta_j}{\eta_i}}}\right) + \frac{C^2}{B^2}\beta\left(\frac{Rq}{C} - \frac{A}{C}\right) + \kappa\bar{\theta}}{\chi + \sum_{k=2}^M \eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa} - qR\right]$$

The goal here is to find the parameters A, B, and C, that ensure market clearing. Given that all private signals conditional on θ are independent, this can be done easily enough by imposing market clearing given this policy function together with the restriction that every agent invests the same η^* as follows:

[21] If we substitute in later expressions for A, B, and C, it is straightforward to observe that this demand curve is always downsloping. This follows from the risk-aversion of the investors and is worth noting as Pavan et al. (2022) argue that the sign of the slope of the demand curve can impact the efficiency properties of a similar model.

$$\alpha a =$$

$$\mathbb{E}_i\left[\chi\tilde{\xi}_i + \sum_{k=2}^{M}\eta_{s_i(k)}\tilde{z}_{s_i(k)} + \hat{\eta}_i \left(\frac{\tilde{z}_i - \iota\tilde{z}_j\sqrt{\frac{\eta_j}{\eta_i}}}{1-\iota\sqrt{\frac{\eta_j}{\eta_i}}}\right) + \frac{C^2}{B^2}\beta\left(\frac{Rq(a,\theta)}{C} - \frac{A}{C}\right) + \kappa\bar{\theta} - Rq(a,\theta)\left(\chi + \sum_{k=2}^{M}\eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa\right)|\theta, q(a,\theta)\right]$$

If we evaluate this conditional expectation and impose that all $\eta_k = \eta^*$ for any $k \in [0,1]$ (which implies that $\hat{\eta}_i = \eta^*(1-\iota)^2/(1-\iota^2) > \eta^*$) then we arrive at

$$\alpha a = \left(\chi + \left(M - 1 + \frac{(1-\iota)^2}{1-\iota^2}\right)\eta^*\right)\theta + \frac{C^2}{B^2}\beta\left(\frac{Rq(a,\theta)}{C} - \frac{A}{C}\right) + \kappa\bar{\theta} - Rq(a,\theta)\left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^* + \frac{C^2}{B^2}\beta + \kappa\right)$$

$$\implies Rq(a,\theta)\times\left[\left(\frac{C^2}{B^2}\right)\frac{\beta}{C} - \left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^* + \frac{C^2}{B^2}\beta + \kappa\right)\right] = \alpha a - \kappa\bar{\theta} - \left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^*\right)\theta + \left(\frac{C^2}{B^2}\right)\frac{\beta A}{C}$$

$$\implies Rq(a,\theta) = \underbrace{\frac{\alpha\bar{a} - \kappa\bar{\theta} + \left(\frac{C^2}{B^2}\right)\frac{\beta A}{C}}{\left(\frac{C^2}{B^2}\right)\frac{\beta}{C} - \left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^* + \frac{C^2}{B^2}\beta + \kappa\right)}}_{A} + \underbrace{\frac{\alpha}{\left(\frac{C^2}{B^2}\right)\frac{\beta}{C} - \left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^* + \frac{C^2}{B^2}\beta + \kappa\right)}}_{B}\times(a-\bar{a})$$

$$+ \underbrace{\frac{-\left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^*\right)}{\left(\frac{C^2}{B^2}\right)\frac{\beta}{C} - \left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^* + \frac{C^2}{B^2}\beta + \kappa\right)}}_{C}\times\theta$$

This gives us three equations: A equals the first term, B equals the second, and C equals the third. Most immediate in the solution is the term $\frac{C}{B}$ since the denominators of these terms cancel leaving

$$\frac{C}{B} = \frac{-\left(\chi + \left(M-1+\frac{(1-\iota)^2}{1-\iota^2}\right)\eta^*\right)}{\alpha} \implies \frac{C^2}{B^2}\beta = \rho_\theta = \frac{I\hat{M}^2\beta}{\alpha^2} \tag{A.2}$$

This term happens to be market efficiency as well given how we interpret the price as a signal. Thus Equation A.2 immediately establishes that Proposition 2 holds in any equilibrium, as it is quadratic in the (always positive) information mesh.

Given its pivotal role, it will be convenient for us to substitute in $I\hat{M}$ for the information mesh terms from here on, i.e., With this, we can solve easily for the third term, C, i.e.,

$$
\begin{aligned}
C &= \frac{-I\hat{M}}{\left(\frac{I\hat{M}^2}{\alpha^2}\right)\frac{\beta}{C} - \left(I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta + \kappa\right)} \\
\implies -I\hat{M} &= \left(\frac{I\hat{M}^2}{\alpha^2}\right)\beta - \left(I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta + \kappa\right)C \\
\implies C &= \frac{I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta}{I\hat{M} + \kappa + \frac{I\hat{M}^2}{\alpha^2}\beta} \\
\implies C &= \left[1 + \kappa\left[I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta\right]^{-1}\right]^{-1}
\end{aligned}
$$

Notice that C is also increasing in the information mesh. This is relevant for the analysis in Section 4.3. Further, with C in hand, we can solve for B

$$
\begin{aligned}
B &= \frac{-\alpha}{I\hat{M}}C \\
\implies B &= -\frac{\alpha}{I\hat{M}} \times \left[1 + \kappa\left[I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta\right]^{-1}\right]^{-1} \\
\implies B &= -\alpha\left[I\hat{M} + \kappa\left[1 + \frac{I\hat{M}}{\alpha^2}\beta\right]^{-1}\right]^{-1}
\end{aligned}
$$

And finally, we can solve for A

$$
\implies A = \frac{\kappa\bar{\theta} - \alpha\bar{a}}{\left(I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta + \kappa\right)}
$$

We have shown now that A, B, and C are uniquely determined for a given η^* (as η^* uniquely determines $I\hat{M}$). Now we can go back to the information acquisition game, from which we can derive the remaining results. We begin by demonstrating a useful lemma:

Lemma 1. *The second-order approximation to Problem 2 is a well-defined problem with a solution governed either (1) by first-order necessary condition combined with an additional sufficiency condition that can always be satisfied or (2) by a corner solution of $\eta_i^* = 0$.*

Proof. We begin by deriving the second-order approximation to the objective function assuming that investor i builds on source $s_i(2)$'s information. We build on the insights of Proposition 5 and express the problem in terms of $\hat{\eta}_i = (\sqrt{(1-\iota^2)\eta_i} - \sqrt{\iota^2 \eta_{s_i(2)}})^2$. Note that the agent takes $\eta_{s_i(2)}$ as given.

$$
\begin{aligned}
&\mathbb{E}_i \left[-\exp\left(-\alpha \tilde{c}_i\right)\right] \\
&\quad = \mathbb{E}_i \left[-\exp\left(-\alpha \left[w_0 R + \tilde{b}_i \times \left(\tilde{\theta}_i - \tilde{q}R \right) \right] \right) \right] \\
&\quad = -\exp\left(-\alpha w_0 R\right) \mathbb{E}_i \left[-\exp\left(-\alpha \times \frac{\left(\mathbb{E}_i \left[\tilde{\theta}_i | \tilde{q}, \tilde{\xi}_i, \{\tilde{z}_{s_i(k)}\}_{k=1}^M \right] - \tilde{q}R \right) \left(\tilde{\theta}_i - \tilde{q}R \right)}{\alpha Var_i(\tilde{\theta}_i | \tilde{q}, \tilde{\xi}_i, \{\tilde{z}_{s_i(k)}\}_{k=1}^M)} \right) \right] \\
&\quad = -\exp\left(-\alpha w_0 R\right) \mathbb{E}_i \left[-\exp\left(\begin{array}{c} -\underbrace{\left[\chi + \sum_{k=2}^{M} \eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa \right]}_{\text{“}P(\hat{\eta}_i)\text{''}} \times \\ \left[\frac{\chi \tilde{\xi}_i + \sum_{k=2}^M \eta_{s_i(k)} \tilde{z}_{s_i(k)} + \hat{\eta}_i \left[\frac{\tilde{z}_i - \iota \tilde{z}_j \sqrt{\eta_{s_i(2)}/\eta_i}}{1 - \iota \sqrt{\eta_{s_i(2)}/\eta_i}} \right] + \frac{C^2}{B^2}\beta \left[\frac{\tilde{q}R}{C} - \frac{A}{C} \right] + \kappa \bar{\theta}}{\chi + \sum_{k=2}^M \eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa} - \tilde{q}R \right] \times \\ \left[\tilde{\theta} - \tilde{q}R \right] \end{array} \right) \right]
\end{aligned}
$$

Where the last line builds on the fact that investor i's signal is derived by taking the difference with the signal from $s_i(2)$. From here, we add zero to the second term in the product embedded in the exponential function by adding $-\tilde{\theta}P(\hat{\eta}_i)/P(\hat{\eta}_i) + \tilde{\theta}$. This simplifies the expression significantly by turning all of the signals in the numerator of this term into iid innovations instead.

$$
\begin{aligned}
&\mathbb{E}_i \left[-\exp\left(-\alpha \tilde{c}_i\right)\right] \\
&\quad = -\exp\left(-\alpha w_0 R\right) \mathbb{E}_i \left[-\exp\left(\begin{array}{c} -P(\hat{\eta}_i) \times \\ \left[\frac{\chi \tilde{\nu}_i + \sum_{k=2}^M \eta_{s_i(k)} \tilde{\epsilon}_{s_i(k)} + \hat{\eta}_i \tilde{\hat{\epsilon}}_i + \frac{C^2}{B^2}\beta \times \frac{B}{C}[\tilde{a} - \bar{a}] + \kappa(\bar{\theta} - \tilde{\theta})}{P(\hat{\eta}_i)} + \tilde{\theta} - \tilde{q}R \right] \times \\ \left[\tilde{\theta} - \tilde{q}R \right] \end{array} \right) \right]
\end{aligned}
$$

Now, if we plug in the equilibrium linear pricing expression $qR = A + B(a - \bar{a}) + C\theta$ and pull shock volatilities out of the innovations such that all shocks become standard and independent normals (denoted by sn), we finally derive an expression for welfare given by

$$\mathbb{E}_i\left[-\exp\left(-\alpha\tilde{c}_i\right)\right]$$
$$= -\exp\left(-\alpha w_0 R\right)\mathbb{E}_i\left[\exp\left(\begin{array}{c}\left[A+\frac{B}{\sqrt{\beta}}sn(\tilde{a})-(1-C)\left(\bar{\theta}+\frac{1}{\sqrt{\kappa}}sn(\tilde{\theta})\right)\right]\times\\ \left[-\left(A+\frac{B}{\sqrt{\beta}}sn(\tilde{a})-(1-C)\left(\bar{\theta}+\frac{1}{\sqrt{\kappa}}sn(\tilde{\theta})\right)\right)\times P(\hat{\eta}_i)+\right.\\ \left.\sqrt{\chi}sn(\tilde{\nu}_i)+\sum_{k=2}^{M}\sqrt{\eta_{s_i(k)}}sn(\tilde{\epsilon}_{s_i(k)})+\sqrt{\hat{\eta}_i}sn(\tilde{\epsilon}_i)+\sqrt{\kappa}sn(\tilde{\theta})\right]\end{array}\right)\right]$$

In this form, the objective function is primed and ready for a second-order approximation over all the (now) iid, standard normal shocks. The tremendous convenience of this approach will be that all linear and cross-partial terms will disappear, leaving only the constant and some second-order (variance) terms in the final expression.

Taking a second-order approximation around the mean of the standard normal innovations and following this by taking of an expectation over them yields a relatively simple expression, i.e.,

$$\begin{aligned}\mathbb{E}\left[\hat{U}_i(\eta_i)\right] &= -\exp\left(-\alpha w_0 R\right)\times\exp\left(-\left[A-(1-C)\bar{\theta}\right]^2 P(\eta_i)\right)\times \\ &\left[1-\frac{B}{\sqrt{\beta}}\left(\frac{B}{\sqrt{\beta}}P(\eta_i)-\frac{C}{B}\sqrt{\beta}\right)-\frac{1-C}{\sqrt{\kappa}}\left(\frac{1-C}{\sqrt{\kappa}}P(\eta_i)+\sqrt{\kappa}\right)\right]\end{aligned} \tag{A.3}$$

where

$$P(\eta_i)=\chi+\sum_{k=2}^{M}\eta_{s_i(k)}+\eta_i\frac{(1-\iota)^2}{1-\iota^2}+\frac{C^2}{B^2}\beta+\kappa$$

is a measure of the total precision with which the agents infer the fundamental asset value. Notice that we substitute in η_i for $\hat{\eta}_i$, which will allow us to later impose the symmetry restriction that η is the same for all agents. $P(\eta_i)$ is linear in η_i, which is under the agent's control. Also, it is noteworthy that the signal constructed by η_i is generally more precise as a result of the negativity of ι, i.e., the agent learns more from his own signal than he does from the others in his network *precisely because he is allowed to build on one of these sources.*

This objective function is non-concave, but nevertheless it is easy to show that first-order conditions are necessary and, with one additional requirement, sufficient. To see this observe first the first-order condition

$$\frac{\partial \mathbb{E}\left[\hat{U}_i\right]}{\partial \eta_i} = \frac{(1-\iota)^2}{1-\iota^2}\exp\left(-\alpha w_0 R\right)\exp\left(-\left[A-(1-C)\bar{\theta}\right]^2 P(\eta_i)\right)\times \underbrace{\left[\left[A-(1-C)\bar{\theta}\right]^2\left(2C-\left(\frac{B^2}{\beta}+\frac{(1-C)^2}{\kappa}\right)P(\eta_i)\right)+\left(\frac{B^2}{\beta}+\frac{(1-C)^2}{\kappa}\right)\right]}_{X(P(\eta_i))}$$

This expression is strictly positive provided the that we call $X(P(\eta_i))$ is positive. $X(P(\eta_i))$ is linear and decreasing, which implies that either

1. If $X(P(0)) \leq 0$, then the objective function is always decreasing and the corner solution of $\eta_i = 0$ is optimal.
2. If $X(P(0)) > 0$, then there will be some threshold, $P' \geq 0$ where $X(P') = 0$ and beyond which marginal utility becomes negative and, thus, the objective decreasing. Thus, we know that the investor will always choose something less than this P'.

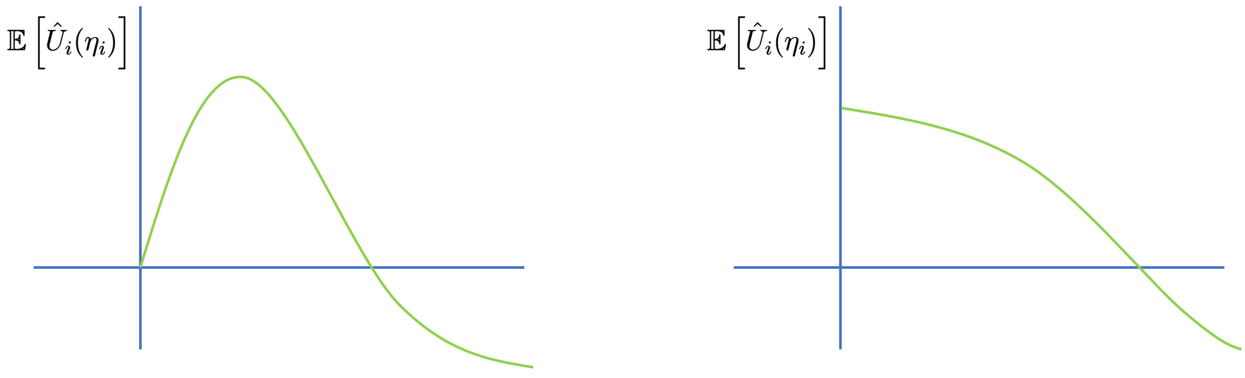
In the case of a possible interior solution, the second-order condition for this problem becomes relevant and is given by

$$\frac{\partial \mathbb{E}\left[\hat{U}_i\right]^2}{\partial^2 \eta_i} = -\exp\left(-\alpha w_0\right)\left(\frac{(1-\iota)^2}{1-\iota^2}\right)^2\times\exp\left(-\left[A-(1-C)\bar{\theta}\right]^2 P(\eta_i)\right)\times \left[\left[A-(1-C)\bar{\theta}\right]^2\left[X(P(\eta_i))+\left(\frac{B^2}{\beta}+\frac{(1-C)^2}{\kappa}\right)\right]\right]$$

It is clear to see that the problem will also be concave so long as the objective is increasing, i.e., $X(P(\eta_i)) \geq 0$.

Intuitively, the objective function must resemble one of the shapes given in Figure 6. The condition that $X(P(\eta_i)) \geq 0$ implies that the objective is increasing at that particular η_i. The case of Figure 6a (combined with the fact that marginal costs at $\eta_i = 0$ are zero by assumption) will deliver an interior solution while the case of Figure 6b will deliver a corner solution at zero (despite the fact that marginal costs are zero at $\eta_i = 0$).

Since the cost function is convex, this implies that a first-order condition is both necessary and sufficient



(a) $X(P(0)) \geq 0$ **(b)** $X(P(0)) < 0$

Figure 6: Possible Equilibrium Objectives as Functions of η_i

for optimality provided we add one additional condition. Namely that the η_i^* that satisfies

$$\frac{\partial \mathbb{E}\left[\hat{U}_i\right]}{\partial \eta_i}(\eta_i^*) = \lambda C'(\eta_i^*) \qquad \text{(A.4)}$$
$$\text{and } X(P(\eta_i^*)) \geq 0$$

If it is ever the case the $X(P(\eta_i)) < 0$ for all $\eta_i \geq 0$, then the objective function is always decreasing in η_i and the solution will be the corner solution of $\eta_i = 0$. But if there is some interval on which this term is positive, then the solution will be interior and unique and governed by the conditions above.

□

In our analyses, we will restrict attention the non-degenerate cases for which the solution is governed by Equations A.4. In particular, equilibrium levels of information acquisition will be those that satisfy this expression when $\eta_{s_i(k)} = \eta^*$ for $k = 1, \ldots, M$.

Numerically, we compute equilibria as follows. First, we check whether $X(P(0)) \leq 0$. If it is, then the only possible equilibrium is $\eta^* = 0$ and the equilibrium is unique.

If $X(P(0)) > 0$, we use a non-linear solver to find values of η^* that satisfy the first expression in Equation A.4. If the solution also satisfies the second expression, then we are done. If not, then we search on the smaller area between zero and the previous η^*. By the intermediate value theorem and the concavity of $\hat{U}$ and the convexity of C in this region, a unique η^* must exist.

This procedure also establishes the uniqueness of the competitive equilibrium. We have already established that the pricing function is unique conditional on η^*, and now we have established that, fixing all fundamentals, the equilibrium value of η^* must also be unique.

A.3 Proof of Proposition 3

Proof. We will show this by deriving a formula relating the variance of the log-returns to private information precision. We'll begin by multiplying the return variance by $1/R^2$, which will not affect this response but will prove convenient for derivations.

$$
\begin{aligned}
Var\left(\log\left(\frac{\tilde{\theta}}{q\tilde{R}}\right)\right) &= Var\left(-\log\left(1+\left(\frac{A}{\tilde{\theta}}+\frac{B(\tilde{a}-\bar{a})}{\tilde{\theta}}+C-1\right)\right)\right) \\
&\approx Var\left(\frac{A}{\tilde{\theta}}+\frac{B(\tilde{a}-\bar{a})}{\tilde{\theta}}+C-1\right) \\
&\approx Var\left(\frac{A}{\tilde{\theta}}+\frac{B(\tilde{a}-\bar{a})}{\tilde{\theta}}\right)
\end{aligned}
$$

Where the penultimate line follows from (1) the first-order approximation that $\log(1+x)\approx x$ when x is close to zero, which it will be since $\theta/(qR)$ will be gross excess returns in the neighborhood of one and (2) the fact that $Var(X)=Var(-X)$. The final line follows from the fact that $C-1$ is constant. From this point, by the law of total variance we can express this as

$$
\begin{aligned}
Var\left(\log\left(\frac{\tilde{\theta}}{q\tilde{R}}\right)\right) &\approx \mathbb{E}\left[Var\left(\frac{A}{\theta}+\frac{B(\tilde{a}-\bar{a})}{\theta}|\tilde{\theta}=\theta\right)\right]+Var\left(\mathbb{E}\left[\frac{A}{\theta}+\frac{B(\tilde{a}-\bar{a})}{\theta}|\tilde{\theta}=\theta\right]\right) \\
&\approx \frac{B^2}{\beta}\mathbb{E}\left[\frac{1}{\tilde{\theta}^2}\right]+A^2Var\left(\frac{1}{\tilde{\theta}}\right)
\end{aligned}
$$

Assumption 1 is sufficient to ensure that the derivative of B with respect to $I\hat{M}$ is positive. Because $B<0$, this implies that as the information mesh thickens, B^2 attenuates toward zero, which reduces volatility.

To see this, note that

$$
\frac{\partial B}{\partial I\hat{M}} = \frac{\alpha}{\left(I\hat{M}+\frac{\kappa}{1+\frac{I\hat{M}}{\alpha^2}\beta}\right)^2}\times\left[1-\frac{\frac{\kappa\beta}{\alpha^2}}{\left(1+\frac{I\hat{M}}{\alpha^2}\beta\right)^2}\right]
$$

which will be positive provided

$$\left(1+\frac{I\hat{M}}{\alpha^2}\beta\right)^2 > \frac{\kappa\beta}{\alpha^2}$$

which will always be true under Assumption 1, as it ensures that the right-hand side is less than one while the left-hand side is always greater than one.

Assumption 2 simply eliminates the impact of the constant A term. $\square$

A.4 Proof of Proposition 4

To get at the correlation across investor portfolios, we look at the covariance of their forecasts for the fundamental, θ, as (1) all investors have the same forecast errors despite getting different signals and (2) conditional on having the same forecast error, investment strategies differ only by $\mathbb{E}_i\left[\tilde{\theta}\right]$. Thus, we're looking at the following covariance for i and j, neither of which is another's source nor do they share any common sources (as this would happen with probability zero in a random draw).

$$Cov\left(\mathbb{E}_i\left[\tilde{\theta}\right], \mathbb{E}_j\left[\tilde{\theta}\right]\right) =$$

$$Cov\left(\frac{\chi\xi_i + \sum_{k=2}^{M}\eta_{s_i(k)}z_{s_i(k)} + \hat{\eta}_i\hat{z}_i + \frac{C^2}{B^2}\beta\left(\frac{Rq}{C}-\frac{A}{C}\right)+\kappa\bar{\theta}}{\chi + \sum_{k=2}^{M}\eta_{s_i(k)} + \hat{\eta}_i + \frac{C^2}{B^2}\beta + \kappa}, \frac{\chi\xi_j + \sum_{k=2}^{M}\eta_{s_j(k)}z_{s_j(k)} + \hat{\eta}_j\hat{z}_j + \frac{C^2}{B^2}\beta\left(\frac{Rq}{C}-\frac{A}{C}\right)+\kappa\bar{\theta}}{\chi + \sum_{k=2}^{M}\eta_{s_j(k)} + \hat{\eta}_j + \frac{C^2}{B^2}\beta + \kappa}\right)$$

$$= \frac{1}{(I\hat{M}+\frac{C^2}{B^2}\beta+\kappa)^2} Cov\left(\chi\xi_i + \sum_{k=2}^{M}\eta_{s_i(k)}z_{s_i(k)} + \hat{\eta}_i\hat{z}_i + \frac{C^2}{B^2}\beta\frac{Rq}{C}, \chi\xi_j + \sum_{k=2}^{M}\eta_{s_j(k)}z_{s_j(k)} + \hat{\eta}_j\hat{z}_j + \frac{C^2}{B^2}\beta\frac{Rq}{C}\right)$$

$$= \frac{1}{(I\hat{M}+\frac{C^2}{B^2}\beta+\kappa)^2}\left(\frac{C^2}{B^2}\beta\right)^2 Var(\frac{Rq}{C}) = \frac{\frac{C^2}{B^2}\beta}{(I\hat{M}+\frac{C^2}{B^2}\beta+\kappa)^2}$$

where $\hat{z}_i$ denotes investor i's signal that he constructs via a difference between his signal and that of his best source, which features a noise component that is orthogonal to all other shocks in the economy.

In equilibrium, we have an expression for the precision of the price (when viewed as a signal). Plugging this in, we get

$$Cov\left(\mathbb{E}_i\left[\tilde{\theta}\right],\mathbb{E}_j\left[\tilde{\theta}\right]\right)=\frac{\frac{(I\hat{M})^2}{\alpha^2}\beta}{(I\hat{M}+\frac{I\hat{M}^2}{\alpha^2}\beta+\kappa)^2}$$

Now if we normalize by the variance of each of these forecasts, we get the desired correlation across portfolios:

$$Corr\left(\mathbb{E}_i\left[\tilde{\theta}\right],\mathbb{E}_j\left[\tilde{\theta}\right]\right)=\frac{I\hat{M}^2}{\alpha^2}\beta$$

Thus the correlation across two investment portfolios follows the same trend as the rest of the economy. In fact, it's actually exactly equal to price informativeness, which makes sense. Since the only point of contact for those not networked together is the price, two randomly selected investment portfolios will look more alike whenever prices are more informative, which is whenever aggregate private information collection goes up.

A.5 Proof of Proposition 6

We consider first the case in which the equilibrium is interior and then consider the possible corner case of $\eta^*=0$. We begin by writing out explicitly the key equilibrium condition, i.e., Equation A.4, without the X-positivity restriction. This looks as follows.

$$\left(\frac{1-\iota^2}{(1-\iota)^2}\right)\lambda C'(\eta^*)=$$
$$\underbrace{(1-C)^2\bar{\theta}^2\exp\left(-(1-C)^2\bar{\theta}^2P\right)\times\left[2C-\left(\frac{B^2}{\beta}+\frac{(1-C)^2}{\kappa}\right)P\right]+\exp\left(-(1-C)^2\bar{\theta}^2P\right)\left[\frac{B^2}{\beta}+\frac{(1-C)^2}{\kappa}\right]}_{RHS}$$

where

$$P=I\hat{M}+\frac{I\hat{M}^2}{\alpha^2}\beta+\kappa$$

To demonstrate this result for the interior case, we need only show that this expression dictates that as M increases $I\hat{M}$ must also increase. This expression is rather complicated as B and C are also dependent on $I\hat{M}$ and will move in accordance with this expression as M increases. As such, it will be most convenient to differentiate these sub-expressions and collect terms. We denote the derivative of an object, x, with respect to M by $d_M x$. We begin with the information mesh itself.

$$d_M I\hat{M} = \left(M - 1 + \frac{(1-\iota)^2}{1-\iota^2}\right) d_M \eta^* + \eta^*$$
$$\implies d_M \eta^* = \frac{d_M I\hat{M} - \eta^*}{M - 1 + \frac{(1-\iota)^2}{1-\iota^2}}$$

The ultimate object of interest for us is $d_M I\hat{M}$ and the last line will allow us to substitute this into $d_M \eta^*$, which will appear on the marginal cost side of the equilibrium condition.

Fortunately, the terms P, B, and C only move with M insofar as the information mesh moves. This allows us to express their differentiation in M entirely through the differentiation in $I\hat{M}$ (as well as exogenous parameters, of course), which is incredibly convenient when the expressions are this convoluted. We do so as follows. First for the P-term:

$$d_M P = \left[1 + 2I\hat{M}\frac{\beta}{\alpha^2}\right] d_M I\hat{M}$$

and the C-term:

$$d_M C = \frac{\frac{\kappa}{\left(I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta\right)^2}\left[1 + 2I\hat{M}\frac{\beta}{\alpha^2}\right]}{\left(1 + \frac{\kappa}{I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta}\right)^2} d_M I\hat{M}$$

and the B-term:

$$d_M B = \left(\frac{\alpha C}{I\hat{M}^2} - \frac{\alpha}{I\hat{M}} \frac{\frac{\kappa}{\left(I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta\right)^2}\left[1 + 2I\hat{M}\frac{\beta}{\alpha^2}\right]}{\left(1 + \frac{\kappa}{I\hat{M} + \frac{I\hat{M}^2}{\alpha^2}\beta}\right)^2}\right) d_M I\hat{M}$$

The linearity of $d_M I\hat{M}$ in each of these terms implies that when we differentiate the equilibrium condition we arrive at

$$\left(\frac{1-\iota^2}{(1-\iota)^2}\right)\lambda C''(\eta^*) \times \frac{d_M I\hat{M} - \eta^*}{M - 1 + \frac{(1-\iota)^2}{1-\iota^2}} = d_{IM} RHS \times d_M I\hat{M}$$
$$\left(\frac{\left(\frac{1-\iota^2}{(1-\iota)^2}\right)\lambda C''(\eta^*)}{M - 1 + \frac{(1-\iota)^2}{1-\iota^2}} - d_{IM} RHS\right) d_M I\hat{M} = \frac{\left(\frac{1-\iota^2}{(1-\iota)^2}\right)\lambda C''(\eta^*)}{M - 1 + \frac{(1-\iota)^2}{1-\iota^2}} \eta^*$$

From this expression we can observe the result. The right-hand side of this expression is clearly positive.

Thus, $d_M I\hat{M}$ will be positive provided the term in the parentheses on the left-hand side is also positive. The cost-term clearly is by the convexity of C. Thus, the result holds as long as $d_{IM}RHS$ is small and/or negative.

While $d_{IM}RHS$ is incredibly convoluted, there are a few things we can say about it. First, if the pricing terms (B and C here) were fixed, then $d_{IM}RHS$ would be negative. This follows from the convexity of the objective at the interior as the information mesh is a simple linear transformation of the choice variable (η_i).

Second, as χ acts as an exogenous lower bound on the information mesh, we can increase it to increase the value of $I\hat{M}$. If we do so, it is clear from the expressions for $d_M C$ and $d_M B$ that these derivatives go to zero. Thus, if χ is large, then these equilibrium pricing effects become negligible. These two effects ensure that $d_{IM}RHS$ is either negative or only mildly positive.

The result is established then by realizing that we can increase λ to ensure that the first (cost-based) term is always larger than this small-to-negative $d_{IM}RHS$ term, which is independent of λ.

The preceding argument established the monotonicity of $I\hat{M}$ in M when the equilibrium was interior. Corner solutions are even easier to see. If the equilibrium is at a corner, then the marginal cost of information acquisition is larger than the marginal benefit, not equal to it. As M increases, the marginal cost will remain constant at 0, as M will not directly affect the cost. Also, as M increases, the marginal benefit will remain the same. This is because $\eta^* = 0$ implies that $I\hat{M}$ will not change with M. Since $I\hat{M}$ does not change with M, neither do P or any of the pricing coefficients. This implies that if the solution is a corner for *any* M, it is a solution *for all* M. Thus, $I\hat{M}$ is flat in M, which implies that it is (weakly) monotone in it.

It is worth noting that the restrictions on λ and χ are sufficient but not necessary. In all parameterizations we sampled, we find the information mesh to be increasing in M.

B Additional Material

B.1 Example Parameters

For the example in Sections 4.2 and 4.3, we set $\alpha = 2.0$, $\bar{a} = 2.0$, $\beta = 1.0$, $\kappa = 2.0$, $\bar{\theta} = 2.0$, $p = 2.0$, and $\lambda = 1.0$. In Section 4.2, we also set $\chi = 5.0$ (this parameter is varied and studied in its own right in Section 4.3). These parameters satisfy Assumptions 1 and 2, but they are not special in ways besides those examined in the paper, i.e., all alternate parameterizations that we explored that also satisfy Assumptions 1 and 2 share the same basic properties.

B.2 Social Planner's Problem

We begin by returning to Equation A.3, which is the objective function for the investors in the information-gather stage. There are several relevant externalities at play here that the social planner accounts for that private investors do not.

The social planner's problem addresses these externalities by choosing all η-terms simultaneously (rather than taking the others as given and choosing one) *and* by substituting in the equilibrium expressions for A, B, and C.

Making these substitutions, the social planner's problem is given by

$$\begin{aligned}
&\mathbb{E}\left[\hat{U}_{SP}(\eta)\right] \qquad\qquad \text{(A.5)}\\
&= -\exp\left(-\alpha w_0 R\right) \times \exp\left(-\left[1-C(\eta)\right]^2 \bar{\theta}^2 P_{SP}(\eta)\right) \times \left[2C(\eta) - \left[\frac{B(\eta)^2}{\beta} + \frac{(1-C(\eta))^2}{\kappa}\right] P_{SP}(\eta)\right]
\end{aligned}$$

where

$$P_{SP}(\eta) = \chi + \left(M - 1 + \frac{(1-\iota)^2}{1-\iota^2}\right)\eta + \frac{\left(\chi + \left(M - 1 + \frac{(1-\iota)^2}{1-\iota^2}\right)\eta\right)^2}{\alpha^2}\beta + \kappa$$

where we make use of the fact that Assumption 2 implies that $A = 0$. The terms $B(\eta)$ and $C(\eta)$ are defined in the proof of Proposition 1.

In contrast to the equilibrium problem, the curvature properties of this objective are much less straightforward, so when we search for a solution to the social planner's problem we search for a global optimum.

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