Notes on Sovereign Debt

Econ 40025: International Macroeconomics

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1 Introduction to Sovereign Debt

In this section, we will focus on markets for sovereign debt and the crises that accompany them. What is sovereign debt? It is debt issued by a government that has the ultimate authority over its repayment, hence we call the debt *sovereign*. For instance, contrary to, corporate debt, there is no legal framework through which creditors can demand repayment e.g. bankruptcy procedures. Sovereign debt has historically come in two primary forms: Bank debt (e.g. Latin America in 70's and 80's) and government bonds (Argentina in late 90's and Eurozone members), though especially with bonds, the contracts can vary widely by issuance.

Two key frictions pervade the market for sovereign debt issuance. The first is the inability of the sovereign to commit to repayment (or to any action benefiting the lenders in the future). The second is the limited capacity of the lenders to expropriate assets in the event of a default; since the borrower is sovereign in nature, there does not exist a legal authority that can enforce repayment of the defaulted-on debts. Given these two hurdles, why do these transactions ever take place? Why would the creditors purchase debt with no credible guarantee of repayment and why would sovereign governments even bother to enter a market so fraught with difficulties?

Sovereign governments may have a multiplicity of reasons for wanting to issue debt to foreign lenders, but we will focus on two. First, the sovereign may have an interest in *smoothing his consumption*. Consumption here can be interpreted explicitly as government consumption or, through strategic domestic tax policy, the consumption of the entire economy. This mirrors exactly the reason why an entire economy may choose to run a current account imbalance in Obstfeld and Rogoff (1996), Chapters 1 and 2: The sovereign may wish to save during good times and borrow during bad times. Changes in the stock of foreign assets held *by the government* is called the **Public Current Account**, and is a subset of the current account. When a government is issuing debt to foreign debt, it is running a public current account deficit, and when it is repurchasing debt (or issuing less short-term debt today than ir repaid today), it is running a public current account surplus.

There is a second reason that a sovereign government may want to borrow, however, and that is to *front-load his consumption*. Governments, particularly those in emerging markets where crises arise, tend to be impatient relative to their foreign creditors. They would rather have the funds now than wait until tomorrow, whereas foreign lenders are happy to wait for returns in the future. In our model from Obstfeld and Rogoff (1996), this would imply that $\beta < \frac{1}{1+r}$.

2 Limited Commitment in Foreign Debt Issuance

2.1 Basic Set-Up

Let us first recall the endowment-economy model in Obstfeld and Rogoff (1996), Chapter 2. We will take it and augment it to include features of Kehoe and Perri (2002). In this environment, we will show that the sovereign's lack of ability to commit to debt repayment will impose a **Debt Limit**, i.e. a level of debt above which the sovereign cannot borrow. Some authors have termed this *credit rationing*, but we will simply think about it as a debt limit.

We start from what we know. Consider our infinite-horizon economy with a deterministic endowment, $\{y_s\}_{s=t}^{\infty}$ from Chapter 2. For simplicity, we will subtract from explicitly incorporating government spending by assuming that the government itself is the one that is choosing the consumption and debt stream. This assumption is easy to relax, but we maintain it for simplicity.

The sovereign government chooses for the households a sequence of consumption, $\{c_s\}_{s=t}^{\infty}$, to maximize

$$\sum_{s=t}^{\infty} \beta^{s-t} u(c_s) \tag{1}$$

subject to a lifetime budget constraint[1] given by

$$-b_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [y_s - c_s] \tag{2}$$

where the stock of debt, which is on the LHS and is positive when $b_t < 0$, must be financed by reducing the NPV of consumption expenditures below the NPV of the endowment stream. Notice that in this simple model, the trade balance in period s is simply given by $tb_s = y_s - c_s$.

Let us denote the solution to this constrained maximization problem by the sequence of constraints $\{c_s^\star(b_t)\}_{s=t}^{\infty}$ for an initial stock of debt, b_t. We can then define the sovereign's *Value Function* in period t by

$$V_t(b_t) = \sum_{s=t}^{\infty} \beta^{s-t} u(c_s^\star(b_t)) \tag{3}$$

Notice that it is necessarily the case that $V_t(b_t)$ is a increasing function, since the more debt the sovereign takes on (lower b_t), the less income the sovereign can devote to consumption and the more he has to devote to the repayment of debt.

2.2 Limited Commitment

We can use this framework to model limited commitment on the part of the sovereign in the following way. Let us suppose that the sovereign cannot promise to repay the stock of debt he issues in period t, $-b_{t+1}$: He may in fact choose to default on it and walk away from his foreign creditors. In the event that this occurs, the lenders employ the harsh strategy of never lending to the sovereign again, which is called autarky. This set-up is in the vein of the classic work of Eaton and Gersovitz (1981). For simplicity of analysis, we will assume that there are no commitment problems after $t+1$ (this assumption can be relaxed).

Since the sovereign cannot interface with foreign lenders in autarky, he is forced to consume his own

[1] Here, we are diverging slightly from the Obstfeld and Rogoff. We will interpret a stock of debt, $b_t < 0$, as the total amount of *payments* that come due in time t i.e. principal plus interest. Obstfeld and Rogoff interpret it as only principal, and thus, they tack on the extra rb_t in interest payments. Our framework is more conducive to sovereign debt analysis, as we will soon see.

endowment in each period. Thus, the value of autarky in any period t is given by

$$V_{A,t} = \sum_{s=t}^{\infty} \beta^{s-t} u(y_s) \tag{4}$$

Notice a couple of interesting things about the value of autarky. First, it is *independent* of the debt stock, $-b_t$: Since the sovereign will never interface with foreign creditors again, he never has to repay them and the size of any outstanding debts is irrelevant. Second, notice that it must be the case that

$$V_{A,t} \leq V_t(0) \tag{5}$$

i.e. a sovereign with zero debt will prefer to have access to credit markets so that he could potentially front-load or smooth his consumption. This result follows because a sovereign with zero debt access to credit markets always *can* choose the autarky allocation. Thus, if he chooses something different, it must provide a higher utility.

The lenders are risk-neutral. This means that they will offer a price schedule that reflects their probability of repayment. In particular, they must be indifferent between investing in a risk-free asset (such as a US Treasury) that provides a return of r, and a potentially risky asset, such as a sovereign bond, which provides a return of $\hat{r} \geq r$, but might be defaulted on. In particular, since $Pr(Repayment) + Pr(Default) = 1$, the following break-even or no-arbitrage condition must hold

$$\begin{aligned} 1 + r &= (1 + \hat{r}_{t+1}) \times Pr(Repayment_{t+1}) + 0 \times Pr(Default_{t+1}) \\ \rightarrow \frac{1}{1 + \hat{r}_{t+1}} &= \frac{Pr(Repayment_{t+1})}{1 + r} \end{aligned}$$

This condition tells us many interesting things about the debt structure of the sovereign contract. First, we can use it to compute the **Spread** on the sovereign bond, which is the difference between what the interest sovereign borrower pays on his debt versus what a risk-free borrower would pay. The spread in our case is simply $s_{t+1} = \hat{r}_{t+1} - r$. If there was no default risk i.e. $Pr(Repayment_{t+1}) = 1$, then the spread would be zero and $\hat{r}_{t+1} = r$. However, it will be positive in the presence of any default risk.

Second, we can define the price of a bond issued in period t as $q_t = \frac{1}{1+\hat{r}_{t+1}}$. If the sovereign wishes issue a bond promising one dollar in $t+1$, lenders will purchase that bond for $q_t < 1$ today.

What is the repayment frequency in our simple case? Lenders know that if they lend the sovereign an amount b_{t+1} such that $V_{t+1}(b_{t+1}) < V_{A,t+1}$, then the sovereign will default and they will not be repaid; however, if b_t is such that $V_{t+1}(b_{t+1}) \geq V_{A,t+1}$, then lenders will be no default.[2] Thus, we can define the threshold **Debt Limit**, $\bar{b}_{t+1}$ by

$$V_{t+1}(\bar{b}_{t+1}) = V_{A,t+1} \tag{6}$$

Notice that the sovereign can issue debt at the risk-free rate when $b_{t+1} \geq \bar{b}_{t+1}$, but that when the debt levels are higher i.e. $b_{t+1} < \bar{b}_{t+1}$, the price will be zero. Notice that, by construction, $\bar{b}_{t+1} \leq 0$. If the sovereign's debt limit is always a true *debt* limit, and will not pertain when the sovereign is attempting to save i.e. $b_{t+1} > 0$.

The analysis is greatly simplified since there are no shocks/randomness in the economy. As a consequence, default and repayment probabilities are either zero or one. In this case, we can define the price of debt, which will be influenced by the size of the debt issuance, by

$$q_t(b_{t+1}) = \begin{cases} \frac{1}{1+r} & , \quad b_{t+1} \geq \bar{b}_{t+1} \\ 0 & , \quad b_{t+1} < \bar{b}_{t+1} \end{cases} \tag{7}$$

This schedule can be seen graphically in Figure 1 (recall that debt is negative assets in this model).

We are now ready to formulate the problem of the sovereign in period t:

$$\max_{b_{t+1}} \; u\left(y_t + b_t - q_t(b_{t+1})b_{t+1}\right) + \beta V_{t+1}(b_{t+1}) \tag{8}$$

Notice that we can exploit the structure of the q_t function to make the problem a bit simpler:

$$\max_{b_{t+1}} \; u\left(y_t + b_t - \frac{1}{1+r}b_{t+1}\right) + \beta V_{t+1}(b_{t+1}) \tag{9}$$
$$\text{subject to} \;\; b_{t+1} \geq \bar{b}_{t+1}$$

[2]We are assuming here if that if the sovereign is indifferent between repaying and defaulting, he always repays.

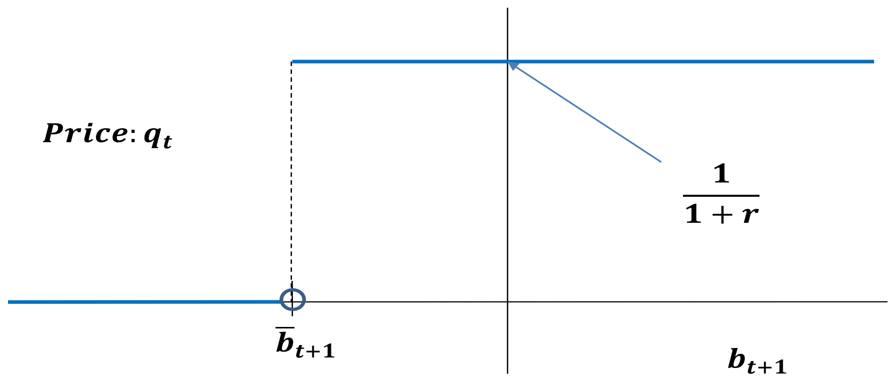

Figure 1: Price Schedule: $q_t(b_{t+1})$

This problem will have a fairly simple solution, which we can compute as follows: Solve the problem from time t without the limited commitment constraint i.e. Problem 9 without the constraint. This is simply the endowment economy from Obstfeld and Rogoff, Chapter 2. Call its solution $\hat{b}^\star_{t+1}$. The full solution will take one of two forms:

1. If $\hat{b}^\star_{t+1} \geq \bar{b}_{t+1}$, then the limited commitment constraint does not bind and incorporating it does not affect the results of Chapter 2 at all. In this case, the optimal $b^\star_{t+1} = \hat{b}^\star_{t+1}$

2. If $\hat{b}^\star_{t+1} < \bar{b}_{t+1}$, then the limited commitment constraint *does bind*. The sovereign will borrow right up to this constraint, and thus the solution will be $b^\star_{t+1} = \bar{b}_{t+1}$.

In the latter case, the sovereign's inability to commit to repayment in the future *prevents him from smoothing/front-loading consumption as much as he would like to today.*

How low (close to zero) is $\bar{b}_{t+1}$? Usually pretty low. Debt limits in these models tend to be very binding and very small. The reason is because the punishment, autarky, is generally not that bad. This point was made by Lucas (1987) with regard to business cycles. Macroeconomic fluctuations tend to embed the optimal response of households to intrinsic fluctuations. Given that these responses are already optimal, the benefit from smoothing out the intrinsic fluctuations is relatively small. Thus, the difference between $V_{t+1}(0)$ and $V_{A,t+1}$ is relatively small, which implies that $\bar{b}_{t+1}$ is close to zero.

2.3 Lessons and Intuition

When is this latter case (the one in which the constraint binds) the more relevant one? Quantitatively quite often, as we will soon find out, but to give some intuition we outline here a couple of general principles

1. **The constraint will tend to bind for low values of β.** This parameter scales up and down the relative value of consumption today, but changes the relative values of repayment and autarky only very slightly, thus leaving the debt limit almost unchanged. Thus, this debt limit places a bound on the amount of consumption *front-loading* that the sovereign can undertake.

2. **The constraint will tend to bind for low values of y_t/high negative values of b_t.** In both cases, the sovereign is faced with a relatively small endowment today that can be allocated toward consumption. The sovereign would like to borrow a lot in this case, but may run into the debt limit quickly. In this case, the debt limit places a bound on the amount of consumption *smoothing* that the sovereign can undertake.

3. **The debt limit is set by the sovereign's willingness to pay, not his ability to pay.** The fact that the sovereign will default even if autarky is only mildly better implies that there are cases in which the sovereign *has the resources* to pay off debts in period $t+1$, he is just not willing to do so. Thus, defaults that would arise in this framework are fundamental in nature, but they are not typically driven by insolvency.

4. **Autarky Punishment/Reputation Concerns can generate very little in terms of sustainable debt.** Autarky alone is not a terribly costly punishment, and so sustainable debt levels under this scheme are pretty low. In other words, high debt levels can be very expensive to service and are typically not worthwhile given the alternative.

Notice that limited commitment *asymmetrically restricts consumption-smoothing*. It prevents the sovereign from borrowing as much as he would like in bad times, but it places no restriction on how much the sovereign can save in good times.

This simple model, while illuminating, is lacking along a couple of relevant dimensions. First, we can never observe *positive* spreads in the model; although it is theoretically possible, the sovereign will never in the optimal solution borrow at a spread any higher than zero i.e. $\hat{r}_{t+1} = r$. Second, it will have extreme difficulty generating debt levels anywhere near what we see in the data, since reputation-based lending in which autarky alone is the punishment generally cannot sustain very high levels of debt. We will attempt to address these problems in the next section.

3 Allowing for 'Fundamental' Default

We now introduce some uncertainty into the previous model. This will allow for us to see some realized default in equilibrium as well as positive spreads. We will undertake this in a fairly simple way, by introducing a variation of a 'default taste shock,' similar to Aguiar and Amador (2013). In particular, we will assume again that the world is characterized by a deterministic endowment, but that there is some uncertainty with regard to 'populist' sentiments in the sovereign economy. In particular, the value of default is now given by

$$V_{d,t+1} = V_{a,t+1} + m_{t+1} \tag{10}$$

where $m_t + 1$ is a random variable with a cumulative-distribution function given by $F(\cdot)$. For those without a statistics background, this simply means that $Pr(m_{t+1} \leq m) = F(m)$. The function $F(\cdot)$ will be increasing and bounded between 0 and 1. We'll assume that it's differentiable, and that $F(m) = 0$ whenever $m \leq \underline{\mathrm{m}}$ and that $F(m) = 1$ whenever $m \geq \bar{m}$. A sample CDF can be seen in Figure 2.

We will assume that this m-shock, while random, has a mean of zero. Thus, the benefit of defaulting is on average the value of autarky. However, political or social forces may push the government to be more or less inclined to pay off outstanding debts. If m_t is really high, then so too is the value of default. This could be interpreted as the election of a populist leader who cares less about the country's reputation with international creditors. We saw a situation like this with the recent Argentinean 'default' of 2014, in which Argentina refused to pay hold-out bondholders from their last default. The motivation was largely political defiance, since Argentina had the means to pay the hold-outs had they desired.

In contrast, when m_{t+1} is low, so too is the value of default. A low value of m_{t+1} could mirror political or social pressure to repay debts for reasons of integrity or reputation. In either case, we'll assume that it

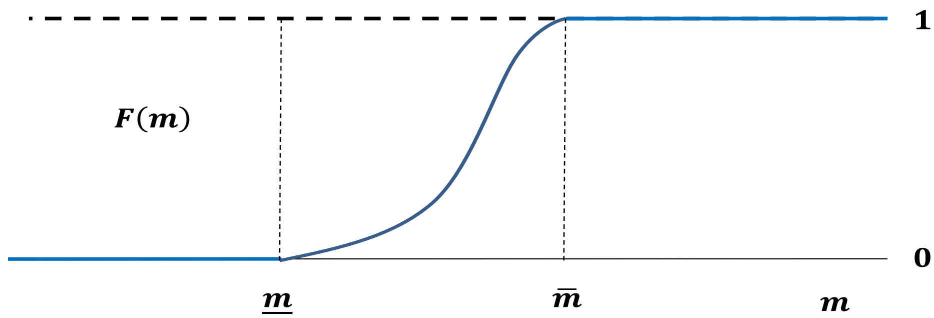

Figure 2: Sample Cumulative Distribution Function: $F(m)$

has nothing to do with the underlying economic fundamentals (y_{t+1}). Nevertheless, its presence will round out the drastic cliff in the q-function that can be seen in Figure 1.

We now assume that the sovereign defaults in $t+1$ whenever $V_{d,t+1} > V_{t+1}(b_{t+1})$. This is the same as saying that he repays whenever $V_{A,t+1} + m_{t+1} \leq V_{t+1}(b_{t+1})$. Notice that we can write the probability of repayment now as

$$\begin{aligned} Pr(Repayment_{t+1}) &= Pr(V_{A,t+1} + m_{t+1} \leq V_{t+1}(b_{t+1})) \\ &= Pr(m_{t+1} \leq V_{t+1}(b_{t+1}) - V_{A,t+1}) \\ &= F(V_{t+1}(b_{t+1}) - V_{A,t+1}) \end{aligned}$$

Thus, we can write the debt price, the parallel of Equation 7, as follows

$$q_t(b_{t+1}) = \frac{F(V_{t+1}(b_{t+1}) - V_{A,t+1})}{1+r} \tag{11}$$

Notice that the two extremes of the default-taste-shock, $\underline{m}$ and $\bar{m}$, will translate to two levels of debt, $\underline{b}_{t+1}$ and $\bar{b}_{t+1}$. For debt levels less than $\underline{b}_{t+1}$, the sovereign will repay with probability one, whereas for debt levels greater than $\bar{b}_{t+1}$, the sovereign will default with probability one. Anything in the interior will generate *some chance* default, but it will not be certain. Thus, the 'cliff' from the pricing schedule in

Figure 1 gets rounded out. This can be seen in Figure 3.

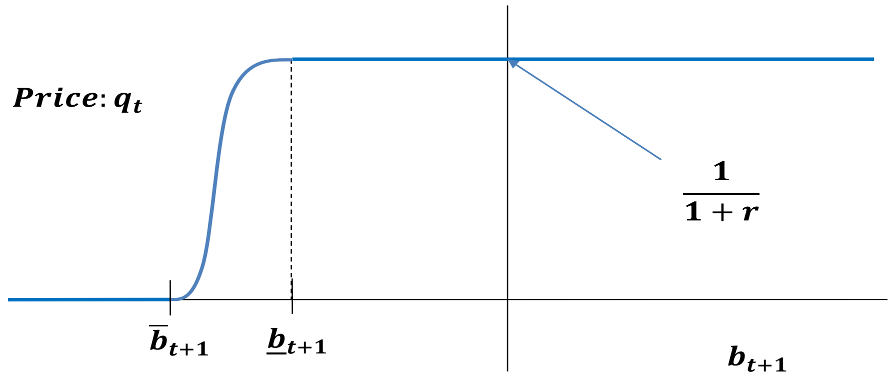

Figure 3: Sample Pricing Schedule: Some Default Possible

With this minor change, we can re-write the sovereign's problem from Equation 8 as follows

$$\max_{b_{t+1}} \; u\left(y_t + b_t - q_t(b_{t+1})b_{t+1}\right) + \beta E_{\tilde{m}_{t+1}}\left[\max\{V_{t+1}(b_{t+1}), V_{A,t+1} + \tilde{m}_{t+1}\}\right] \tag{12}$$

Notice that we have to account for the fact that the sovereign might in fact default tomorrow in the sovereign's preferences. Before, we never needed to account for this, since the sovereign never found default optimal given his debt limits.

This problem will generally (but not always) have a nice, interior solution when $\beta < \frac{1}{1+r}$. Assuming that F is differentiable, we can take a first-order condition to see what it would like:

$$FOC(b_{t+1}) : 0 = -u'(y_t + b_t - q_t(b_{t+1})b_{t+1}) \times [q_t(b_{t+1}) + b_{t+1}q_t'(b_{t+1})] + \ldots \tag{13}$$

The terms further to the right (captured by the dots) are messy and uninformative. The bracketed terms, however, provide a wealth of insight. Notice that is contained of two terms:

$$q_t(b_{t+1}) + b_{t+1}q_t'(b_{t+1})$$

The first term is the additional revenue raised because we issued another unit of debt at a price q_t. The second term is the impact on our *total* revenue because as we issue one more unit of debt, the price drops not just for that one unit but *for the entire stock of debt issued.* The sovereign factors this change in the price into his borrowing decision in exactly the same way that a monopolist factors in that producing more of his good lowers the price on all goods already in his inventory, since the sovereign is essentially a monopolist in this market for sovereign debt. Aguiar and Amador 2014 note that this latter, price-changing term, is quantitatively very important in the sovereign's decision-making.

So what does this imply? It suggests that there are two, relatively orthogonal forces that impact the sovereign's borrowing choice. First, there is the standard consumption-smoothing motive that induces him to borrow in bad times and save in good (apparent in the $u'(\cdot)$ term in Equation 13). Second, there is a price-spread motive: q_t will tend to be low (spreads will be high) in *bad times* and q_t will be high (spreads will be low) in *good times.* Thus, borrowing conditions are *better in good times.* This latter effect induces the sovereign save in bad times and borrow in good times, and flies in the face of consumption-smoothing by generating overly volatile consumption. In practice, most quantitative models generate that the price-spread effect dominates the consumption-smoothing motive, and this causes consumption volatility to be in excess of endowment volatility. This unusual features is actually an empirical regularity in emerging markets (though not in developed nations), and Arellano (2008) has posited that movements in sovereign debt risk may be a key driver of business cycles in emerging markets.[3]

Lastly, note that we can take our framework and map it into a spread on sovereign bond issuance. First, note that the *yield* of a bond is the interest rate that it promises. In our case, the bond yield is simply $\hat{r}_{t+1}$, and it must satisfy $\frac{1}{1+\hat{r}_{t+1}} = q_t(b_{t+1})$. Once we know $q_t(b_{t+1})$, we can compute the yield from $\hat{r}_{t+1} = \frac{1}{q_t(b_{t+1})} - 1$.

Once we have the yield, it is trivial to compute the spread, which is simply the difference between the yield and the risk-free rate, r. In other words, $s_{t+1} = \hat{r}_{t+1} - r = \frac{1}{q_t(b_{t+1})} - (1 + r)$. In this model, a bond will be issued at a strictly positive spread if and only if there is some probability of default in the next period.

[3] Neumeyer and Perri (2005) show that interest rate movements/spreads can generate all of the other comovements right in emerging market business cycles movements.

3.1 Default Costs

So why are spreads high in bad times? Is it clear that the default incentive is greater in bad times? Not from our analysis so far, but this will become clearer once we make one further modification, which will also help us solve our other problem: Realistic debt levels.

We noted earlier that it is hard for models based only on autarkic exclusion to match the external indebtedness anywhere near the levels seen in the data. Thus, creditors must have *some way* of punishing sovereign debtors apart from refusing to lend to them. Two primary theories have been put forward, both of which have roughly the same implication. The first are international sanctions; Mendoza and Yue (2012) argue that once international credit lines are severed to the sovereign, they are also severed to the firms in that country, which can be quite damaging, since foreign lines of credit are often used to finance trade in essential intermediate inputs. The second are banking crises, which tend to occur since domestic banks are often heavily exposed to their own sovereign's debt and so their balance sheet takes a big hit during a default (Sosa-Padilla (2012), Bocola (2014)).

In both cases, the act of defaulting has an immediate impact on domestic output: Either intermediate inputs becomes more expensive or the financial sector loses its ability create liquidity and loanable funds. We can model both in the same way, with a proportional default costs, $\psi \in (0,1)$. Thus, the sovereign's endowment *following default* is now $\{(1-\psi)y_s\}_{s=t}^{\infty}$. This implies that default costs less during bad times than good times, since if $y_l < y_h$, then $\psi y_l < \psi y_h$.

Proportional default costs will drop the value of default, $V_{d,t+1}$, without changing the value of repayment. Thus, the choice to default is now worse and the opportunity cost of repaying debt burdens is higher. As a consequence, higher and more realistic debt levels can be sustained.

Thus, provided the endowment is persistent, spreads will jump during bad times (low y_t). This is because low output today signals low output tomorrow (and thus low default costs and more temptation to default). While this is a nice feature to have, it does imply that it is difficult to determine the causal relationship between sovereign default and recessions. Do defaults cause recessions? Or are they simply the consequence of severe recessions? The answer seems to be somewhere in between: Bad economic conditions make default more attractive, but the default itself tends to exacerbate the situation.

3.2 Lessons and Intuition

1. **Sovereign borrows right up to the 'cliff' in the pricing function.** The front-loading motive is typically large enough that the sovereign creeps up to the edge. This implies that spreads and default frequencies are positive in equilibrium.

2. **The sovereign has a consumption-smoothing motive, but in equilibrium consumption-smoothing typically does not occur.** This is because the consumption-smoothing motive is offset by fluctuations in the equilibrium price of debt, which raises the cost or borrowing in bad times and lowers it in good times.

3. **Sovereign defaults tend to occur in bad times, but the causality is less clear.** Poor economic conditions tend to make default more attractive. However, there are reasons to believe that sovereign defaults cause output drops, amplifying recessions that may have already been there. Thus, empirically it can be very hard to analyze the extent to which bad times were the cause or consequence of a sovereign default.

4 Sentiments and 'Non-Fundamental' Default

In the previous sections, we studied 'fundamental' default, which is a bit of a misnomer since in fact defaults are driven by *willingness* to repay and not *capacity* to repay. Nevertheless, defaults in the model are driven by low output or by shifts in preferences, both of which are fundamental.

We'll shift gears now and explore the different ways that defaults can emerge as coordination failures among agents i.e. shifts in 'market sentiment.' These are defaults that, for the same set of fundamentals in the economy, could have been avoided had there not been a 'pessimism' among investors. In particular, we will explore two types of coordination failures: The first is in which the sovereign's lack of ability to commit to debt issuance leads to excessive borrowing at high spreads; the second is one in which the sovereign faces a liquidity problem, being unable to roll over the short-term debt stock. The latter seems to characterize the Mexican Tequila Crisis of 1994-95 (Cole and Kehoe (1996)) while the former seems a better depiction of the recent Eurozone crisis (Lorenzoni and Werning (2013), Stangebye (2015), Corsetti and Dedola (2013)).

A key feature of 'sentiment'-driven crises in either form is the ability of a credible third-party, such as the central bank in the case of the Eurozone or the US in the case of Mexico, to step in and fix the problem at no cost. Much like deposit insurance can prevent a bank run, if everybody believes that the large third party will purchase debt at the 'good' equilibrium, then private investors will coordinate on that good equilibrium and the crisis will be averted. This seemed to happen in Mexico with the US pledging a backstop in early 1995 and in the Eurozone with Mario Draghi's famous 'whatever it takes' speech in mid-2012.

4.1 Laffer-Curve Multiplicity

There is another relevant sort of confidence-crisis that seems to explain key features of the recent Eurozone crisis. In contrast to many emerging markets-crises, which have the feature that in response to spikes in spreads (drops in q_t), the sovereign reduces his debt position substantially and sometimes starts saving. This is of course because it is expensive to borrow at very high interest rates.

However, in Europe, there was a spike in spreads *but no concomitant deleveraging.* Rather, member nations continued to increase their borrowing and the debt-to-GDP ratios exploded across the board. There is another variation on our model that delivers a crisis of this sort as well that we will explore now.

To do this, we will return to our model of 'fundamental default,' in which default happens with some probability in period $t+1$ according to the realization of some random political/preference shock, m_{t+1}. We will now analyze more closely what exactly the debt auction looks like.

We know that if the sovereign chooses a level of debt, $b_{t+1} < 0$, he must issue this debt at an auction and that debt will have a price $q_t(b_{t+1})$. Thus, auction revenue is given by

$$Rev_t(b_{t+1}) = -q_t(b_{t+1})b_{t+1} \tag{14}$$

We can further say a couple of interesting things about this revenue function. First, we know that $Rev_t(0) = 0$: If the sovereign issues no debt, he will raise no revenue. Second, we know that for small levels of debt ($b' > \underline{b}_{t+1}$), there is no default risk; this implies that $q_t(b') = \frac{1}{1+r}$ and that $Rev_t(b') = \frac{b'}{1+r} > 0$. Finally, we know that if he issues a lot of debt debt ($b'' < \bar{b}_{t+1}$), then he will default for sure tomorrow. In this case, lenders will set the price to zero and $Rev_t(b'') = 0$.

What sort of picture does this paint for our revenue function? It tells us that revenue is *non-monotone* in debt issuance. In other words, revenue is not always increasing in debt issuance; it looks more like a parabola, as in Figure 4. This revenue schedule is often called a **Laffer Curve**.[4] Note that when the sovereign is saving ($b_{t+1} > 0$) we don't have this shape, since there is no default risk and thus the sovereign gets the risk-free price.

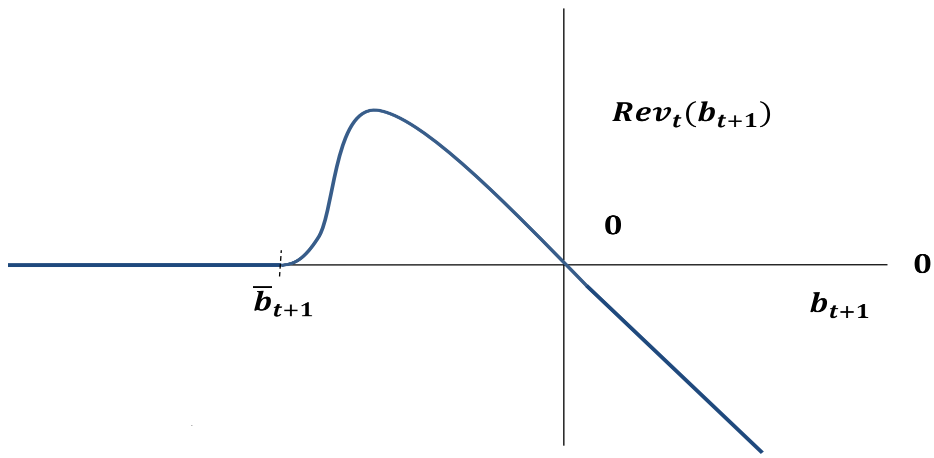

Figure 4: Laffer Curve in Period t

What is the reason for this parabolic curvature? It's really quite simple. The sovereign starts issuing debt at a risk-free rate when debt levels are low. As they climb, however, default risk starts to kick in and the price drops. Eventually the *price starts dropping faster than quantity increases.* At this point, the Laffer curve peaks. This is the maximum possible revenue that the sovereign can attain. After this point, the price drops so fast that additional issuance actually generates less revenue.

What does this suggest for sovereign behavior and why can this lead to a coordination failure? Suppose that the sovereign wanted to raise an amount of revenue, x, at the auction i.e. $x = -q(b_{t+1})b_{t+1}$. This implies that there are *two ways he can do this.* He could issue a small amount of debt at a high price, b_L, or a large amount of debt at a small price, b_H. This can be seen in Figure 5.

Why does this model give us a good picture of the Eurozone crisis? Suppose that we live in a world in

[4]The original Laffer curve described a similar, inverted-U graph with tax revenue as a function of income taxes.

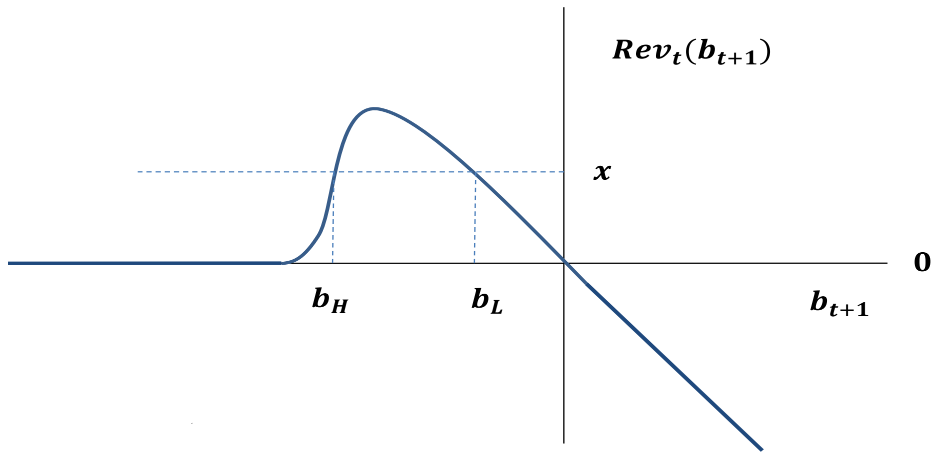

Figure 5: Laffer Curve in Period t

which the sovereign can only commit to a level of *revenue* in any given period and not to an actual level of *debt*. This is not hard to imagine. The governing body goes to their treasury and says: "We need you to issue debt to raise x Euros to fill our budget deficit." The treasury holds a bond auction to issue that debt, but they may not be able to control whether they raise x by issuing b_L or b_H.

We can interpret the Eurozone crisis as a situation in which the sovereign was typically issuing debt on the 'good side' of the Laffer curve, b_L, but suddenly and unexpectedly this shifted to b_H. The key important feature of this shift is that it would have occurred without changing c_t, since $q_t(b_{t+1})b_{t+1}$ would remain the same. This would imply a large increase in debt levels accompanied by a large increase in spreads (fall in q_t) as well as increased default probabilities in $t+1$.

4.2 Rollover Crises

We will now take the model from the previous section and modify it to allow for rollover crises in the vein of Cole and Kehoe (1996). During a roll over crisis, the sovereign has a large stock of debt coming due that he will not be able to service without issuing more debt. So long as lenders issue more short-term debt, the sovereign will not default and will happily continue to make debt payments; however, if the lenders refuse to lend, the sovereign will find himself forced into a state of default.

Why would the lenders not lend? The logic is symmetrical to a bank run. Suppose that you are an individual lender; you are small relative to both the total mass of lenders providing funds to this country and to the country itself. This implies that your actions impact neither the sovereign's decision to default nor the price he receives. Thus, if you don't believe any other lenders will lend to him, then you know for certain that he will default (since he cannot service his existing debt without issuing new debt); thus, you will certainly not want to lend to him, or he will immediately default on you and you will lose your investment. However, if you believe that every other lender is lending to him, then you will lend to him as well, since you know he is perfectly equipped to service his current debt burdens and (probably) any further debt burdens he is taking on in period t.

If every lender has this belief structure, then there can be two equilibria. In the first equilibrium, nobody lends because nobody believes that anybody else will lend. The sovereign's best response in this situation is to default, which fulfills their expectations. In the second equilibrium, everybody lends because everybody believes that everybody else will lend. The sovereign's best response is to repay and roll over hist debt stock and no crisis occurs.

To introduce these crises, we will change one, simple feature: The sovereign can default in period t instead of $t+1$. Thus there is no limited commitment problem new. Instead, the key assumption will be that the default decision in period t will be made *after the sovereign issues debt at the period t auction.* Once $t+1$ rolls around we will keep all assumptions in the previous section i.e. we won't allow for rollover risk or any other kind of default in $t+1$ or any future period for simplicity.

The sovereign now faces one of two problems, depending on the lenders' belief regime. If the lenders believe he will default, they will offer him a price of zero on any new debt issuance, thus he will not issue any new debt. The value to him of repaying *existing debt* is given by

$$\hat{V}_t(b_t) = u\left(y_t + b_t\right) + \beta V_{t+1}(0) \tag{15}$$

If lenders offer him a price of zero in this way, he has the option to default. Doing so would clearly be optimal provided

$$\hat{V}_t(b_t) < V_{A,t}$$

As before, there will be a threshold level of debt at which these two are equal; let us call it $\underline{\mathrm{b}}_t$ i.e. $\hat{V}_t(\underline{\mathrm{b}}_t) = V_{A,t}$. Notice that whenever $b < \underline{\mathrm{b}}_t$, the sovereign will default *if the lenders do not lend to him i.e. offer him a price of zero.* For a level of debt $b \geq \underline{\mathrm{b}}_t$, it is not an equilibrium for the lenders to offer a zero price on the debt because the debt level is such that he will repay. Thus, their belief of default would not be justified.

This 'rollover' default will only be true confidence crisis provided that $V_t(b_t) \geq V_{A,t}$, where

$$V_t(b_t) = \max_{b_{t+1}} \ u\left(y_t + b_t - \frac{1}{1+r}b_{t+1}\right) + \beta V_{t+1}(b_{t+1}) \tag{16}$$

i.e. the value the sovereign receives if lenders were to lend to him. Thus, confidence crises can occur whenever

$$V_t(b_t) \geq V_{A,t} > \hat{V}_t(b_t) \tag{17}$$

since the sovereign will repay and rollover debt obligations when lenders believe that he will repay, but will default on those obligations if they believe that he will default. We can use our original debt limit, which is defined $\bar{b}_t$ according to $V_t(\bar{b}_t) = V_{A,t}$ to see the debt boundaries for which confidence crises arise i.e. debt levels in the range of $\left(-\underline{\mathrm{b}}_t, -\bar{b}_t\right]$.

This model was developed to explain Mexico's 1994 Tequila crisis, since fundamentals in the economy did not seem too bad (reasonable growth rates, fiscal solvency, strong real currency, etc.), nevertheless a panic came about and default nearly occurred.

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