International Macroeconomics
Lecture 1: Benchmark Model

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International Macroeconomics

- Macroeconomics is all about dynamics
  - Distinguishes this course from international trade, international relations, etc.
  - Focus on progression of international economic relationships
    1. Trends and 'normal' international relationships
    2. Crises: Build-ups, events, and aftermaths
  - This class
    1. Study models of international, dynamic, macroeconomic relations
    2. Apply models to real-world events

Definitions

- Traditional Macro Accounting

$$GDP = C + I + G + NX$$
$$GNP = GDP + Net\ Foreign\ Factor\ Payments$$

- GDP/GNP are *flow* measures
- Two very relevant *stock* measures
  1. Domestic assets/capital stock (K_t)
  2. Foreign assets/capital stock (B_t)
- If r is rate of return on foreign assets

$$GDP_t = Y_t$$
$$GNP_t = Y_t + rB_t$$

Examples

- **Assets** are items/objects that generate returns real returns for their owners
- Domestic assets
  1. A manufacturing plant in Detroit owned by GM
  2. A US government bond purchased by JP Morgan
  3. A newly renovated Notre Dame football stadium (owned by ND)
- Foreign assets
  1. A manufacturing plant in Korea owned by Ford
  2. A German bond (*bund*) purchased by AIG
  3. A US toll-road owned and maintained by a French company
- Foreign investment greater than domestic investment abroad $\rightarrow B_t < 0$
  - On the contrary, it must always be $K_t > 0$

Law of Motion

- Y_t is 'income,' while B_t can be thought of as additional assets/debts

- How to determine B_{t+1}? Economy-wide resource constraint

$$\underbrace{Y_t + (1+r)B_t}_{Income} = \underbrace{C_t + I_t + G_t}_{Domestic\ \ Expenditure} + \underbrace{B_{t+1}}_{Acquisition\ \ of\ \ Foreign\ \ Assets}$$

- More on relationship of B_t to NX_t in a bit...

Definitions

- Accounting for international transactions
  - **Current Account Balance:** Change in the value of a country's net foreign assets (2)
- CA, like GDP, is a *flow* variable

$$CA_t = B_{t+1} - B_t$$

where B_t are foreign assets accumulated by the country at the start of period t

  - $CA_t > 0$: Economy is increasing investment abroad more than countries abroad are investing in it
  - $CA_t < 0$: Other economies are increasing investment in the economy more than the economy is investing abroad

International Transactions

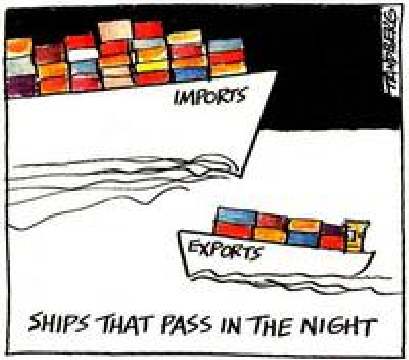
- Don't we already have NX to account for international transactions?
  - Current account finances trade imbalances
- Why does $CA_t \neq NX_t$ (in general)?
  - If $NX > 0$, then goods and services sent abroad must be purchased with *something*
    - Purchased with an increase in foreign asset holdings (claims on foreign production/income)
  - Opposite true when $NX < 0$: Finance the excess of foreign goods and services purchased with claims to own domestic production
  - BUT, foreign claims/liabilities generate returns/require interest above and beyond investment/loan principal (NX)!
  - Thus,

$$CA_t = B_{t+1} - B_t = NX_t + rB_t$$

Terminology

- If the current account balance is non-zero, then we say that **capital is flowing into/out of country**
  - $CA_t > 0 \rightarrow$ capital is flowing *out of* country
  - $CA_t < 0 \rightarrow$ capital is flowing *into* country
- A current account **imbalance** is a loose term that refers to a current account balances in excess of what appears ‘normal’

Imbalanced trade is the *Norm* in most economies



Motivation

- Want a parsimonious model to understand international macroeconomic dynamics
- Understanding this model and extensions can help make sense of real-world phenomena
  1. Why do some countries run deficits while others run surpluses?
  2. What are the long-run consequences of current account imbalances?
  3. How does the current account balance respond to a domestic or foreign shock? How does this differ across shocks?
- Models let us 'play' with toy scenarios to make real-world predictions
- A *Partial Equilibrium Model* takes some prices as given (for us, r)
  - A *General Equilibrium Model* allows for all prices and quantities to fluctuate to meet equilibrium conditions

Basic Model: Households

- Mass of domestic agents live for two periods. Agent i has a utility function:

$$U = u(c_1^i) + \beta u(c_2^i)$$

  where $\beta \in (0, 1)$
- Receive an endowment (GDP) in each period: (y_1^i, y_2^i)
- Can borrow or lend from abroad (or each other) at fixed interest rate: r
- Want to maximize utility by choosing (c_1^i, c_2^i, s^i) subject to

$$\begin{aligned} s^i + c_1^i &= y_1^i \\ c_2^i &= y_2^i + (1 + r)s^i \end{aligned}$$

Solving

- Substitute s^i to arrive at **Lifetime Budget Constraint**

$$c_1^i + \frac{c_2^i}{1+r} = y_1^i + \frac{y_2^i}{1+r}$$

- Substitute c_2^i in objective to get

$$\max_{c_1^i} u(c_1^i) + \beta u\left[(1+r)(y_1^i - c_1^i) + y_2^i\right]$$

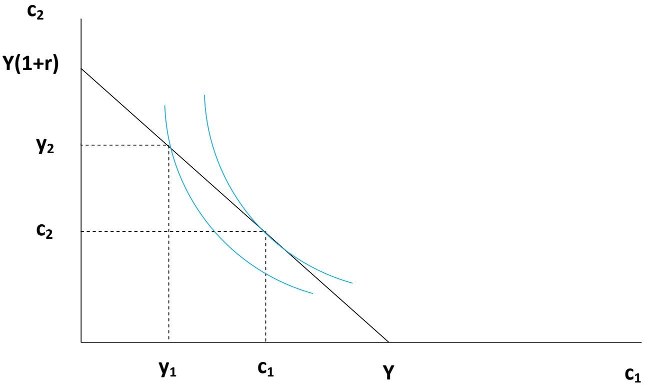
- Assume u is concave: $FOC(c_1^i)$ reveals the **Euler Equation**

$$u'(c_1^i) = (1+r)\beta u'(c_2^i)$$

- If $\beta(1+r) = 1$, then we know that $c_1^i = c_2^i = \bar{c}^i$ where

$$\bar{c}^i = \frac{(1+r)y_1^i + y_2^i}{2+r}$$

Graphical Result: $Y = y_1^i + \frac{y_2^i}{1+r}$



Aggregating

- To make sense of solution, assume all agents are *the same* i.e. homogeneous

$$C_t = c_t^i$$
$$Y_t = y_t^i$$

1. Makes problem tractable
2. Fairly plausible for macro-implications
3. Yields intuitive results

- If all agents are identical, then they won't borrow/lend to each other
  - *All borrowing/lending comes from abroad*

Formulating the Current Account

- Foreign assets, B_t, derived from solution

$$B_1 = 0 \quad \textbf{(Given)}$$
$$B_2 = Y_1 - C_1$$
$$B_3 = 0 \quad \textbf{(Given)}$$

- Recall $CA_t = B_{t+1} - B_t$

$$CA_1 = Y_1 - C_1$$
$$CA_2 = -(Y_1 - C_1)$$

- Notice we can also define from B_t alone

$$CA_t = Y_t + rB_t - C_t$$

Current Account vs. Net Exports

- No I or G (yet), so $NX_t + C_t = Y_t$
- Implies

$$NX_1 = Y_1 - C_1 = CA_1$$

- But

$$NX_2 = Y_2 - C_2$$

$$\rightarrow NX_2 = Y_2 - [Y_2 + (1+r)[Y_1 - C_1]]$$

$$\rightarrow NX_2 = -(1+r)[Y_1 - C_1] \neq CA_2$$

- NX is CA plus returns/interest on assets/liabilities

Determinants of the Current Account: Lesson I

- Assume $\beta(1+r) = 1$. Then $C_1 = C_2 = \bar{C}$ regardless of relative sizes of Y_1 and Y_2
- Two cases:
  1. $Y_1 > \frac{Y_2}{1+r}$: Implies $CA_1 > 0$ and $CA_2 < 0$
     - Surplus in period one and deficit in period two
  2. $Y_1 < \frac{Y_2}{1+r}$: Implies $CA_1 < 0$ and $CA_2 > 0$
- **Current Account is determined by consumption-smoothing behavior**
  - Domestic economy mitigates its own shocks by lending/borrowing abroad
- Note that with B_t, we can also formally distinguish GDP and GNP

$$\begin{aligned} GDP_1 = GNP_1 &= Y_1 \\ GDP_2 &= Y_2 \\ GNP_2 &= Y_2 + r(Y_1 - C_1) \end{aligned}$$

Case Study: Postwar US

- Model predicts Y_t and CA_t should be *positively* correlated
  - Save abroad during good times; borrow from abroad during bad times
- In US,

$$Corr(GDP\ Growth_t, CA_t/GDP_t) \approx .13 > 0$$

Data from St. Louis FRED

- Correlation is not strong (many other explanatory variables), but direction is right

Who's Borrowing and Who's Lending?

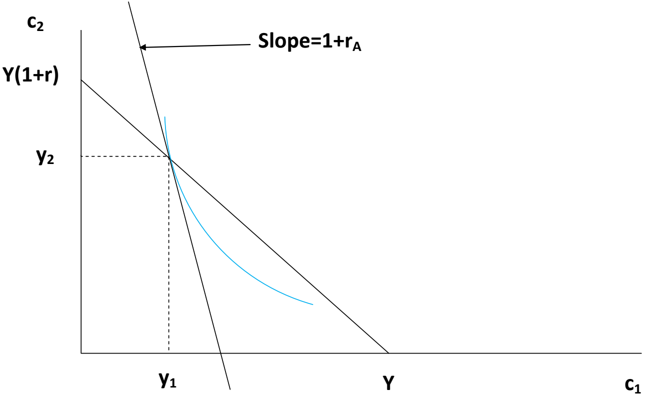
- Suppose no trade abroad is allowed (**Autarky**): Then $C_t = Y_t$ for both periods
- Define the **Autarky Real Interest Rate**, r_A, as the interest rate that would induce $C_t = Y_t$ *even if the country was allowed to borrow/lend abroad* i.e.

$$r_A = \frac{u'(Y_1)}{\beta u'(Y_2)} - 1$$

1. $r > r_A$: The country will be a lender i.e. run a surplus in period one and a deficit in period two
2. $r < r_A$: Opposite occurs
3. $r = r_A$: Current account is zero in both periods

- Follows from concavity of $u(\cdot)$

The Autarky Real Interest Rate



Temporary vs. Permanent Shocks

- Temporary: Y_1 increases but Y_2 stays same
  - Increase CA surplus to transfer extra resources to tomorrow
- Permanent: Y_1 and Y_2 increase/decrease by same amount
  - No change in CA (besides scaling up by magnitude of Y shift)
- Temporary vs. permanent shocks will play a central role in our analysis of emerging markets
- Important to note: *Permanent good shocks do not induce saving*
  - Often they induce opposite: Borrow against higher future income

Adding Government

- Government spends G_t in each period
    - Exogenously specified i.e. not chosen like C_t
- Provides separable utility to households i.e.

$$U = [u(C_1) + \nu(G_1)] + \beta [u(C_2) + \nu(G_2)]$$

- Separability $\rightarrow$ original HH equations still hold
    - Can essentially relabel '$Y_t' = Y_t - G_t$ and proceed as before
- New current account formulation:

$$CA_t = Y_t + rB_t - C_t - G_t$$

- CA moves to smooth fluctuations $Y_t - G_t$ i.e. after-tax income if $G \approx$ gross taxes

Case Study: Postwar US

- Model predicts $\hat{Y}_t = Y_t - G_t$ and CA_t should be *positively* correlated
  - Save abroad when after-tax income high; borrow when low
- In US,

$$Corr(\hat{Y}\ Growth_t, CA_t/\hat{Y}_t) \approx .07 < .13$$

Data from St. Louis FRED

- Again, correlation is not strong (many other explanatory variables), but direction is right
- Weaker than our previous correlation (.13). Why?
  - G_t is *counter-cyclical*
  - Fluctuations in $\hat{Y}_t$ are not as volatile as Y_t: Less saving motive

A Production Economy

- Now, suppose that Y_t must be *produced* using some capital stock i.e.

$$Y_t = F(K_t)$$

  for some production function, $F(\cdot)$
- Assume that $F(0) = 0$ and $F' > 0$ and $F'' < 0$ i.e. diminishing returns
- Capital can be 'eaten' after its used, such that total resources $= Y_t + K_t$
- Capital is an alternative asset to saving abroad $\rightarrow$ Total wealth at end of period t is given by $K_{t+1} + B_{t+1}$
- New capital created with investment I_t

$$K_{t+1} = K_t + I_t$$

A Production Economy: Implications

- **National Saving** is change in domestic wealth

$$S_t = [B_{t+1} + K_{t+1}] - [B_t + K_t] = Y_t + r_t B_t - C_t - G_t$$

- Using the definition of current account and $I_t = K_{t+1} - K_t$, we can deduce

$$CA_t = Y_t + r_t B_t - C_t - G_t - I_t$$

- This implies that

$$CA_t = S_t - I_t$$

  1. If $CA > 0$, then economy is sending some saving abroad
  2. If $CA < 0$, then foreign investors are financing domestic investment

Solving the Production Economy

- How to solve a model with production/determine I_t vs. S_t?
- Given K_1, G_1, G_2, the problem can be written as

$$\max_{C_1, I_1} u(C_1) + \beta u\Bigg(\underbrace{F(I_1 + K_1)}_{F(K_2)=Y_2} + \underbrace{I_1 + K_1}_{K_2} + (1+r)\underbrace{\left[F(K_1) - C_1 - G_1 - I_1\right]}_{B_2} - G_2\Bigg)$$

- Nice feature: $FOC(I_1)$ implies

$$0 = u'(C_2) \times \left[F'(K_2) + 1 - (1+r)\right]$$

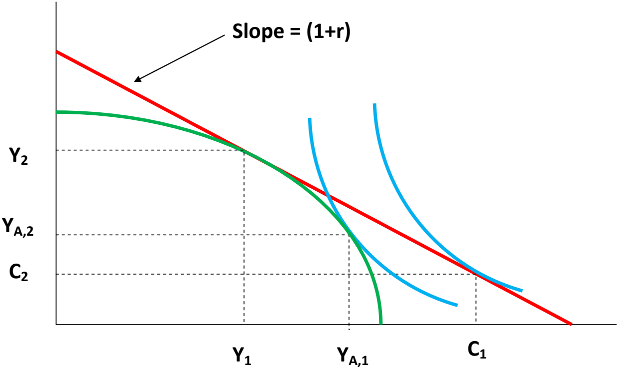
In other words

$$F'(K_2) = r$$
$$\rightarrow I_1 = F'^{-1}(r) - K_1$$

Solving the Production Economy

- Investment completely (and simply) determined from
  1. World interest rate, r
  2. Existing capital stock
- Intuition: $F'(\cdot)$ is the return on capital
  - $F'(K) < r$: Disinvest domestic capital and invest abroad
  - $F'(K) > r$: Borrow from abroad to invest in highly productive domestic capital
- Once I_1 is determined, we get K_2 and Y_2 automatically
  - Solve problem for C_1 as before $\rightarrow$ Arrive at same Euler Equation
- Intuitively
  1. Pick I_1 to maximize the budget set from international exchange at a given r
  2. Tangency condition on international budget set determines C

The Production Economy



Green: PPF
Red: International Borrowing/Lending
Here, $Y_2 = F(K_2) + K_2$ and $Y_1 = F(K_1) + K_1 - K_2$

Determinants of the Current Account: Lesson Two

1. **A CA deficit can result from high-return domestic investment**
2. **A CA surplus can result from low-return domestic investment**

- Wealth chases high-return projects, whether domestic or abroad
- Case Study: Latin America in the 1970's
  1. Several large new oil fields discovered but little infrastructure in place to mine them
  2. Such a productive opportunity → Enormous glut of foreign investment
  3. Years of CA deficits induced heavy indebtedness → Crises in early 80's

Partial Equilibrium Analysis: Summary

- In simple model, current account balance driven by
  1. Smoothing consumption over income fluctuations
  2. Smoothing consumption over after-tax income fluctuations
  3. Geographic location of productive investments

Toward General Equilibrium

- Partial equilibrium analysis took r as given. How is it determined?
- Need a market for international borrowing/lending: r adjusts to clear this market
  - Consider a two-country world: Home and Foreign ($\star$)
  - Start with an endowment economy, no government
- All world resources must be consumed in every period (market clearing):

$$\underbrace{Y_t + Y_t^\star}_{Global\ \ Resources} = \underbrace{C_t + C_t^\star}_{Global\ \ Consumption}$$

- Using $S_t = Y_t - C_t$, this implies

$$S_t + S_t^\star = 0 \quad \rightarrow CA_t + CA_t^\star = 0$$

since $I_t = 0$ in endowment economy

Macro/General Equilibrium Refresher

1. Each country acts *exactly* as in partial equilibrium world (takes r as given)
2. Derive optimal consumption and CA as functions of r: $C_t(r), CA_t(r)$
3. Solve for the r that clears the world market i.e. r_{eq} given be

$$Y_t + Y_t^\star = C_t(r_{eq}) + C_t^\star(r_{eq})$$

Implications/Benefits

1. Neither country controls the global interest rate
2. Both countries are optimizing
3. No global market imbalances (excess supply/demand)

Solving the Global Equilibrium: Saving Functions

- Euler Equation governs saving

$$u'(Y_1 - S(r)) = (1+r)\beta u'(Y_2 + (1+r)S(r))$$

- Totally differentiate to deduce $\frac{dS(r)}{dr}$

$$-u''(C_1(r))\frac{dS(r)}{dr} = \beta u'(C_2(r)) + (1+r)^2 u''(C_2(r))\frac{dS(r)}{dr}$$

which implies

$$\frac{dS(r)}{dr} = \frac{-\beta u'(C_2(r))}{u''(C_1(r)) + (1+r)^2 u''(C_2(r))} > 0$$

- Optimal saving function is *increasing* in r
- Holds for $S^\star$ as well

Solving the Global Equilibrium: Autarky

- Suppose that $r_A^\star = r_A$
- We know that $S(r_A) = S^\star(r_A^\star) = 0$
  - Thus, if $r_{eq} = r_A^\star = r_A$, then
$$S(r_{eq}) + S^\star(r_{eq}) = 0$$

- Global market clearing satisfied $\rightarrow$ Global Equilibrium
  - No country benefits from borrowing/lending to other country
  - $\rightarrow$ No country borrows/lends

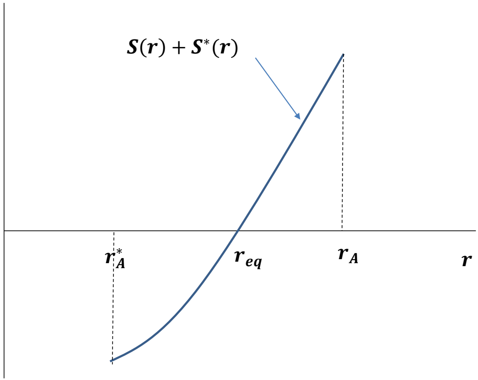
Solving the Global Equilibrium: Interaction

- Suppose that $r_A^\star < r_A$
- Increasing saving function implies two things:
  1. $S(r_A^\star) + S^\star(r_A^\star) < 0$
  2. $S(r_A) + S^\star(r_A) > 0$
- Intermediate Value Theorem tells us that $r_{eq} \in (r_A^\star, r_A)$ exists such that

$$S(r_{eq}) + S^\star(r_{eq}) = 0$$

- Since saving is an increasing function, r_{eq} is unique as well
- Implications
  1. Equilibrium interest rate *must* lie between autarky rates
  2. Country with higher autarky interest rate becomes borrower
  3. Movements in autarky interest rates likely shift r_{eq}

Global Equilibrium



Autarky Interest Rates

- Autarky interest rates determine current accounts and flows of capital (S) across borders
- What determines them? Recall

$$r_A = \frac{u'(Y_1)}{\beta u'(Y_2)} - 1$$

- Note that, by concavity, $u'(\cdot)$ is a *decreasing* function
  1. High Y_2 relative to Y_1 $\rightarrow$ High r_A
  2. Low Y_2 relative to Y_1 $\rightarrow$ Low r_A
- Intuitive: Countries with low (high) relative income today become borrowers (lenders)

Bells and Whistles: Government

- Can trivially add back in government expenditure as before:
- Autarky interest rate becomes

$$r_A = \frac{u'(Y_1 - G_1)}{\beta u'(Y_2 - G_2)} - 1$$

- New lesson: Large government expenditures lower effective income
- **Case Study: World War I**
    - Warring countries incur large government expenditures $\rightarrow$ pushes up r_A
    - Pushes up r_{eq} $\rightarrow$ larger capital flows from non-warring countries
    - Wars are 'financed' from abroad

Wold War I

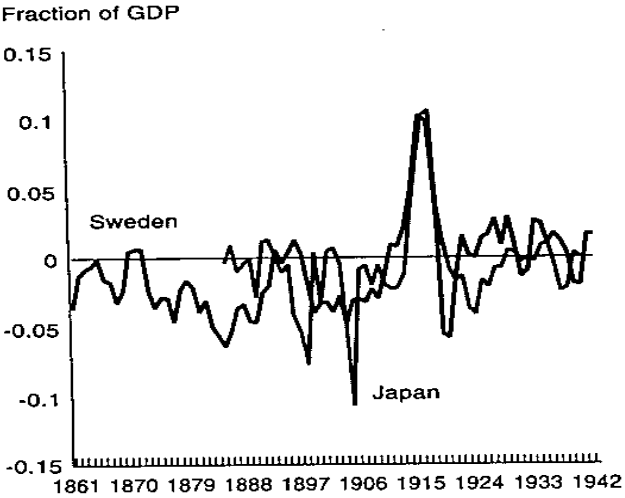


Figure 1.6
Current accounts of Japan and Sweden, 1861–1942

Bells and Whistles: Investment and Production

- Rather than being an endowment, return to $Y_t = A_t F(K_t)$
  - A_t is a measure of productivity/technology
  - K_1 is given
  - K_2 is chosen through I_1: $K_2 = K_1 + I_1$
- World Equilibrium:

$$\underbrace{Y_1 + Y_1^\star}_{World\ \ Supply} = \underbrace{C_1 + C_1^\star + I_1 + I_1^\star}_{World\ \ Demand}$$

- Can be relabeled

$$\underbrace{[S_1(r) - I_1(r)]}_{CA_1} + \underbrace{[S_1^\star(r) - I_1^\star(r)]}_{CA_1^\star} = 0$$

Saving and Investment

- To figure out how CA behaves as a function of r, return to I
- Recall that I_1 is chosen such that

$$r = A_1 F'(K_1 + I_1(r))$$

  i.e. marginal cost of funds from abroad (r) matches marginal benefit of investment (RHS)
- Totally differentiate to get

$$I'(r) = \frac{1}{A_1 F''(K_1 + I_1(r))} < 0$$

- Investment falls as r increases (opportunity cost of funds)

Saving and Investment

- Since $I(r)$ is decreasing and $S(r)$ is increasing, it must be that

$$CA(r) = S(r) - I(r)$$

  is increasing in r
- The autarky interest rate is a bit more complicated, but is easily defined as

$$CA_1(r_A) = 0$$

$$CA_1^\star(r_A^\star) = 0$$

  which immediately implies

$$S_1(r_A) = I_1(r_A)$$

$$S_1^\star(r_A) = I_1^\star(r_A)$$

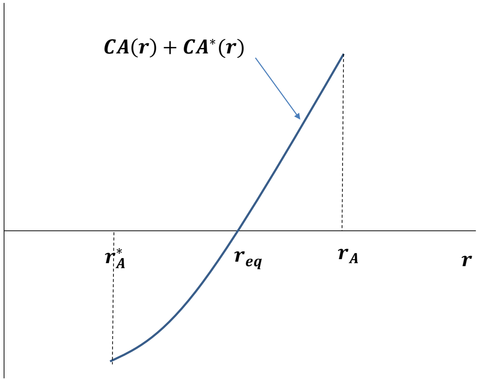
Global Equilibrium

- Again, if $r_A = r_A^\star$, then the only equilibrium is that interest rate and autarky
- Similarly, if $r_A^\star < r_A$
  1. $CA_1(r_A^\star) + CA_1^\star(r_A^\star) < 0$
  2. $CA_1(r_A) + CA_1^\star(r_A) > 0$
- Again, Intermediate Value Theorem implies a unique $r_{eq} \in (r_A^\star, r_A)$ exists such that

$$CA_1(r_{eq}) + CA_1^\star(r_{eq}) = 0$$

- A world equilibrium in which both countries are optimizing along consumption **and** investment decisions *without* consulting each other

Global Equilibrium



Global Equilibrium: Productivity Shifts

- Consider an increase in A, where $Y_t = AF(K_t)$
- Recall that $r_A = AF'(K_1 + S_1) - 1$, since in autarky, $I_1 = S_1$
    1. A increases productivity equally today and tomorrow $\rightarrow$ No large change in saving
    2. Increase in A directly increases r_A
- Rise in autarky interest rate raises r_{eq}
    1. Attracts funds from abroad $\rightarrow$ Foreign ($\star$) invests in productive technology i.e. I_1 increases
    2. Domestic country runs a CA deficit...without even a borrowing motive!
    3. (Over time) accumulate large foreign debts

Global Equilibrium: Applicability

- General, global equilibrium can provide some useful insight, but is limited in its usefulness
- Empirically, world seems more aptly described by
    1. Closed developed economies in general equilibrium
    2. Open emerging markets in partial equilibrium
- Central banks in developed countries determine international r (at least at short maturities)
- Emerging market fluctuations have little to no impact on r

Government Intervention

- Given that world looks like divergence between large, developed economies and small, emerging ones, can developed economies use this?
- Government of developed economy can exploit its size and influence
  1. Government does not behave like competitive agents i.e. not price-taker
  2. Government only cares about domestic households
- By First Welfare Theorem (which applies), the 'laissez-faire' economy was efficient
- Usually get a different allocation. Implies
  1. Home (developed) better off
  2. Foreign (emerging) worse off

Home Government's Problem

- Assume $u(C) = \ln(C)$ (solve in class to derive $S(r)$)
- Using taxes on foreign borrowing, home government can force households to choose C_1, C_2, and r subject to
  1. Foreign saving function, over which Home government has no control

$$S_1^\star(r) = Y_1^\star - C_1^\star(r) = \frac{\beta^\star}{1+\beta^\star} Y_1^\star - \frac{1}{(1+\beta^\star)(1+r)} Y_2^\star$$

  2. Global resource constraint

$$C_1 + C_1^\star(r) = Y_1 + Y_1^\star$$

  3. Home budget constraint

$$C_2 = Y_2 - (1+r)(C_1 - Y_1)$$

Simplifying Home Government's Problem

- Use constraints to boil down to one choice variable
  1. Substitute (1) into (2) to isolate $1 + r$

$$1 + r = \frac{Y_2^\star}{(1 + \beta^\star)(Y_1 - C_1) + \beta^\star Y_1^\star}$$

  2. Substitute above into (3) to arrive at

$$C_2 = Y_2 + \frac{Y_2^\star}{(1 + \beta^\star)(Y_1 - C_1) + \beta^\star Y_1^\star}(Y_1 - C_1)$$

- This last expression is called the Foreign **Offer Curve**

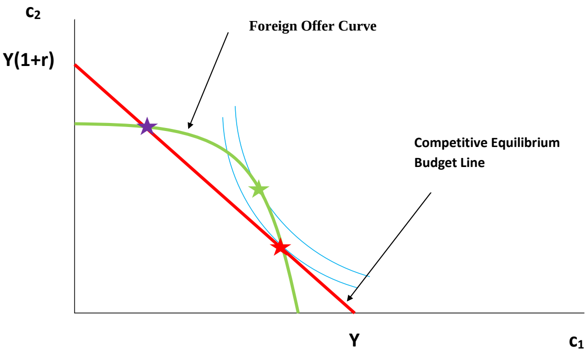
Home Government's Simplified Problem

$$\max_{C_1} u(C_1) + \beta u(C_2)$$

$$\text{s.t.} \quad C_2 = Y_2 + \frac{Y_2^\star}{(1+\beta^\star)(Y_1 - C_1) + \beta^\star Y_1^\star}(Y_1 - C_1)$$

- Unlike competitive equilibrium, solution is generally *not efficient*
  - Another allocation exists which helps foreign without hurting home

Global Equilibrium with Active Home Government



Red Star: Competitive Equilibrium
Green Star: Solution
Purple Star: Autarky

Implementation

- Implement allocation with a tax on foreign borrowing τ (tariff)
  - Rebate proceeds of tax lump-sum back to households
- If green star is to be chosen by competitive individuals and r_{eq} is equilibrium interest rate
  1. At green star, we must have balanced trade
  $$Y_2 - C_2 = (1 + r_{eq})(C_1 - Y_1)$$
  tells us r_{eq}
  2. At green star, domestic households must behave optimally given prices
  $$MRS(C_1, C_2) = -(1 + r_{eq} + \tau)$$
  tells us τ

Empirical Implication

- Model predicts that government ought to better 'terms of trade' for home
  1. When Home wants to save (output high), raise tariffs to increase effective r
  2. When Home wants to borrow (output low), lower tariffs to lower effective r
- Tariffs should be *pro-cyclical*
- Suggestive empirical evidence that this is true: Lake and Linask (2015)
  - Some disagreement here

Stretching Out the Time Horizon in PE World

- 2-Periods is a bit restrictive: Intuition of model applies more generally
- Start with T periods, with time-separable preferences

$$U_t = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+1}) + \cdots + \beta^T u(C_{t+T})$$

- Budget constraint in period t

$$Y_t + (1+r)B_t = C_t + G_t + I_t + B_{t+1}$$

- Definition from before still applies

$$CA_t = B_{t+1} - B_t = Y_t + rB_t - C_t - G_t - I_t$$

Lifetime Budget Constraint

- Notice that

$$B_t = \frac{C_t + G_t + I_t - Y_t}{1+r} + \frac{B_{t+1}}{1+r}$$

Same is true for B_{t+1}, so we can write

$$B_t = \frac{C_t + G_t + I_t - Y_t}{1+r} + \frac{1}{1+r}\left[\frac{C_{t+1} + G_{t+1} + I_{t+1} - Y_{t+1}}{1+r} + \frac{B_{t+2}}{1+r}\right]$$

- Notice that, since the game ends, we must have $B_{t+T+1} = 0$
- Repeat procedure until we hit period $t + T$ i.e.

$$(1+r)B_t = \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s + G_s - Y_s)$$

Lifetime Budget Constraint

$$(1+r)B_t = \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s + G_s - Y_s)$$

- This is the **Lifetime Budget Constraint**
- Present value of stream of expenditures minus present value of stream of income
  1. Can be as large as currently accumulated assets B_t and returns if $B_t > 0$
  2. Must be sufficiently negative so as to pay off outstanding debts and interest if $B_t < 0$
- Both B_t and K_t are given

Solving

- Substitute period budget constraint to get rid of C_t and I_t
- Recall that $Y_t = A_t F(K_t)$ and $K_{t+1} = K_t + I_t$

$$U_t = \sum_{s=t}^{t+T} u\Bigg(\underbrace{(1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s}_{C_s}\Bigg)$$

- Can separately choose B_{s+1} and K_{s+1}:

$$\text{FOC}(K_{s+1}) : A_{s+1} F'(K_{s+1}) = r$$

$$\text{FOC}(B_{s+1}) : u'(C_s) = (1+r)\beta u'(C_{s+1})$$

- Capital allocated where it is most productive
- Foreign assets bought or sold to smooth consumption after optimal investment

Solving: Annuity Values

- Assume that $\beta = \frac{1}{1+r}$. In this case, Euler equation says

$$u'(C_s) = u'(C_{s+1}) \ \ \forall s$$

$$\rightarrow C_s = C_{s+1} = \bar{C} \ \ \forall s$$

- Substituting this into lifetime BC implies

$$\bar{C} = \left[\frac{1}{1-(1+r)^{-(T+1)}}\right]\left(\frac{r}{1+r}\right) \times$$

$$\left[(1+r)B_t + \sum_{s=t}^{t+T}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - G_s - I_s)\right]$$

- Lifetime income governs consumption in any period; not current income

Taking the Limit

- Want to take limit as $T \to \infty$
- Can easily write preferences:

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$$

- Solved *exactly the same way* and implies *exactly the same* first-order conditions
- Difference: Terminal condition...Can no longer use $B_{t+T+1} = 0$

The No-Ponzi Condition

- Natural guess: $\lim_{T\to\infty} B_{t+T+1} = 0$ . . . Incorrect!
- Counterexample: Suppose $\beta = 1/(1+r)$ and $Y_s = \bar{Y}$ for every s

$$\bar{C} = rB_t + \bar{Y}$$

  Consume endowment in every period plus returns on initial assets (minus interest on initial debt)
- Implies $B_s = B_t$ for all s (constantly roll over same stock of assets/debt)
    - Thus, $\lim_{T\to\infty} B_{t+T+1} = B_t \neq 0$
- Real condition is *No-Ponzi Condition:*

$$\lim_{T\to\infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0$$

The No-Ponzi Condition

- Why is this? Take BC of T-horizon case to infinity *without imposing* $B_{t+T+1} = 0$

$$(1+r)B_t = \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (C_s+I_s+G_s-Y_s)+\left(\frac{1}{1+r}\right)^{T} B_{t+T+1}$$

- Last term going to zero necessary and sufficient for present debt/assets to be serviced by lifetime surpluses/deficits
  - Foreigners would never allow debt to grow greater than PV of lifetime surpluses
  - Domestic residents would never consume less than PV of lifetime assets
- Debt can remain positive and even *grow to infinity*, but it must grow at a rate smaller than r
  - When this happens, NPV of debt goes to zero

Sustainability of Debt

- Almost every country has a large stockpile of debt: At what point does this become unsustainable?
- Useful tool: The **Trade Balance** is the *net amount* transferred to foreigners each period

$$TB_s = Y_s - C_s - I_s - G_s$$

- Also known as *Net Exports*. Notice that, from infinite-horizon BC

$$-(1+r)B_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} TB_s$$

Sustainability of Debt

- Trade balance is *not the current account*

$$TB_s = Y_s - C_s - I_s - G_s$$

$$CA_s = Y_s - C_s - I_s - G_s + rB_s$$

- If $B_s < 0$ i.e. country is indebted, then

$$CA_s < TB_s$$

- It is even possible that debt is sustainable when

$$CA_s < 0 < TB_s$$

Can occur without violating TVC if Y grows at $g > r$

Examples

Countries that have frequently run persistent CA deficits sustainably

1. Canada
2. Australia
3. US

A Useful Tool

- Before wrapping up, introduce a useful tool from infinite-horizon analysis
- The *Permanent Level* of a variable X on date t is given by

$$\tilde{X}_t = \frac{r}{1+r} \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} X_s$$

- Intuition: Instead of the (potentially volatile) stream X_s, we get a constant value $\tilde{X}_t$ forever with same NPV i.e. it is true that

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} X_s = \sum_{s=t}^{\infty} \left(\frac{1}{1+r} \right)^{s-t} \tilde{X}_t$$

Applying the Useful Tool

- The infinite sum, $\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$, shows up quite a bit
- Can simplify notation with permanent levels: Illuminating
- Example

$$CA_t = B_{t+1} - B_t = (Y_t - \tilde{Y}_t) - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t)$$

- Highlights two big factors influencing CA
  1. Consumption smoothing (Y_t and G_t)
  2. Particularly productive investment (I_t)
- If none of these things differ from permanent levels, no need to generate an imbalance

The Benchmark Model: Summary

- International macroeconomics can almost be seen as an alternative to trade
- International economic transactions and tariffs motivated by
  1. Insurance motive
  2. Location of productive investments
  3. Monopoly concerns of large nations
- Gains from international exposure arise from ability to hedge domestic shocks and best exploit productive technologies/foreign saving
  - Stands in contrast to canonical models of trade: Comparative advantage, relative resource abundance, increasing returns to scale, etc.