International Macroeconomics
Lecture 4: Limited Commitment

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  - Central bank commits to a monetary trajectory
  - Governments/households commit to repaying foreign debt

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    - Still optimal to adhere to a hard monetary peg *when a crisis actually rolls around*?
- Countless real-world examples of these and many others
    1. Mexican devaluation of 1994
    2. Argentine default (and devaluation) of 2001
    3. Greek default of 2012
    4. ...

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- Necessarily, these models will need to be solved using **backward induction**
  - Only when optimal behavior tomorrow is known can we solve today's problem

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- This set-up is much more tractable
- Basic lessons here hold up in the world where there is never commitment

Example 1: Sovereign Debt

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- **Sovereign debt:** Debt issued by a government that has ultimate authority over its repayment
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- Historically came in two forms: Bank debt and bonds
- Examples
  - US Treasury bonds
  - Argentine government bonds
  - Bank loans to Mexican sub-national governments
- Contracts vary widely and significantly across countries/time

Sovereign Debt

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- Motivations for trade
    1. *Consumption smoothing*: Sovereign may want to use foreign borrowing to smooth out domestic shocks
    2. *Consumption front-loading*: Sovereign may be more impatient than lenders

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- Trade balance $tb_s = y_s - c_s$
- Implement consumption choices through borrowing

$$c_s + \frac{1}{1+r} b_{s+1} = y_s + b_s$$

Basic Environment

- Solution: Combine Euler equation and lifetime BC
- Suppose we have a solution $\{c_s^{\star}(b_t)\}_{s=t}^{\infty}$
- Define new object: A *Value Function* is the lifetime utility attained by implementing the optimal solution i.e.

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- Notice: $V_t(b_t)$ is an increasing function
  - More debt sovereign has (lower b_t)...
  - Less income he can devote to income
  - More income must be devoted to debt repayment over lifetime

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- Sovereign defaults in $t+1$ if

$$V_{A,t+1} > V_{t+1}(b_{t+1})$$

Characterizing Default

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   - $V_{t+1}(b_{t+1})$ increasing in b_{t+1}
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3. It must be the case that

$$V_{A,t+1} \leq V_{t+1}(0)$$

   - Same financial position: Autarky and zero debt
   - Better to have zero debt and access to financial markets
   - Autarky allocation *feasible* but likely not *optimal* with credit market access

Lenders

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- If interest rate on defaultable bond is $\hat{r}_{t+1}$

$$1 + r = (1 + \hat{r}_{t+1}) \times Pr(Repayment_{t+1}) + 0 \times Pr(Default_{t+1})$$

$$\implies \underbrace{q_t}_{Bond\ \ price} = \frac{1}{1 + \hat{r}_{t+1}} = \frac{Pr(Repayment_{t+1})}{1 + r}$$

Lenders

- Things to note...
  1. If no default risk i.e. $Pr(Repayment_{t+1}) = 1$, then

$$r_{t+1} = \hat{r}_{t+1}$$

  2. If this is not the case, then

$$s_{t+1} = \hat{r}_{t+1} - r_{t+1} > 0$$

  where s_{t+1} is the *spread* on the bond

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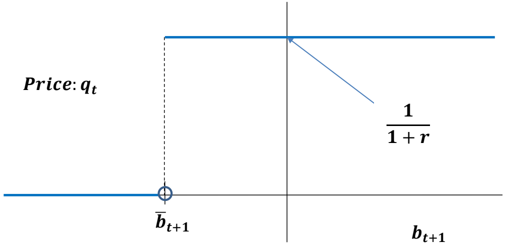
  - If $b_{t+1} \geq \bar{b}_{t+1} \implies$ Repay
- $\bar{b}_{t+1}$ is the **debt limit**
  - Issue below: Get risk-free rate
  - Cannot issue above (infinite interest rate)

Price Schedule



Sovereign Problem

- Sovereign chooses debt issuance by solving

$$\max_{b_{t+1}} u\left(y_t + b_t - q_t(b_{t+1})b_{t+1}\right) + \beta \times \max\{V_{t+1}(b_{t+1}), V_{A,t+1}\}$$

- Features
  1. Sovereign chooses debt issuance taking lender demand as given i.e. *monopolist*
  2. Sovereign cannot *control* default decision tomorrow, but he *knows whether it will happen and accounts for it*

Simplifying

- Sovereign would never borrow past limit (no benefit)
- Problem same as adding a new constraint to the commitment model

$$\hat{V}_t(b_t) = \max_{b_{t+1}} \; u\left(y_t + b_t - \frac{1}{1+r} b_{t+1}\right) + \beta V_{t+1}(b_{t+1})$$

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- Notice this is the same as the commitment model ($V_t(b_t)$) with a borrowing constraint
- It immediately follows that

$$\hat{V}_t(b_t) \leq V_t(b_t)$$

i.e. lack of commitment can only hurt the sovereign

Solving

- This equivalence also implies solution technique
  1. Solve commitment model (i.e. Euler equation and resource constraint)
  2. Check if optimal $b^{\star}_{t+1} \geq \bar{b}_{t+1}$
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  - Low β (relative impatience/consumption front-loading)
  - Low y_t/high negative b_t (recession/debt crisis)

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- *Asymmetrically* restricts consumption smoothing
  - Can save as much as he likes in booms
  - Cannot borrow through recessions

Other lessons

1. Debt limit set by *willingness to pay*, not ability
2. Autarky alone generally gives $\bar{b}_{t+1}$ close to zero
    - Typically need other costs to see large amounts of debt

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- Limitations
    - No default in equilibrium
    - No positive spreads in equilibrium
    - Implied debt levels nowhere near data

Example 2: Maintaining a Peg

- sd

Back to Sovereign Debt

- Allow for uncertainty between debt issuance and repayment decision
- Assume that the value of default is

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where m_{t+1} is a *random variable*, whose value is not known in period t

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- High realization of m_{t+1} may imply default where a low realization would imply repayment

Digression: Random Variables

- Substantial theory behind random variables
- All we'll need is the *Cumulative Distribution Function (CDF)* of the shock m_{t+1}

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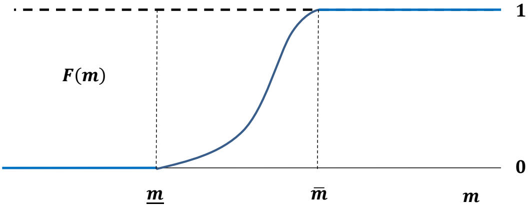
  - CDF completely and fully characterizes randomness associated with shock
  - $F(\cdot)$ increasing function bounded in $[0, 1]$
  - Assume $m_{t+1} \in [\underline{m}, \bar{m}]$ i.e. bounded

$$\implies F(\underline{m}) = 0, \quad F(\bar{m}) = 1$$

Sample CDF



Implications

- Assume also that
  1. CDF is given and everybody knows it
  2. CDF is continuous and differentiable
  3. $E_t[m_{t+1}] = 0$ i.e autarky is average punishment

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  1. CDF is given and everybody knows it
  2. CDF is continuous and differentiable
  3. $E_t[m_{t+1}] = 0$ i.e autarky is average punishment
- Sovereign repays whenever

$$V_{t+1}(b_{t+1}) \geq V_{D,t+1} = V_{A,t+1} + m_{t+1}$$

- Implies

$$Pr(Repayment_{t+1}) = Pr(V_{A,t+1} + m_{t+1} \leq V_{t+1}(b_{t+1}))$$

$$= Pr(m_{t+1} \leq V_{t+1}(b_{t+1}) - V_{A,t+1})$$

$$\implies Pr(Repayment_{t+1}) = F(V_{t+1}(b_{t+1}) - V_{A,t+1})$$

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  2. Two thresholds:

$$\bar{b}_{t+1} : \quad V_{t+1}(\bar{b}_{t+1}) = V_{A,t+1} + \underline{m}_{t+1}$$

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$$\underline{\mathrm{b}}_{t+1} : \quad V_{t+1}(\underline{\mathrm{b}}_{t+1}) = V_{A,t+1} + \bar{m}_{t+1}$$

  3. No longer a ‘cliff’; rounded out in $[\bar{b}_{t+1}, \underline{\mathrm{b}}_{t+1}]$

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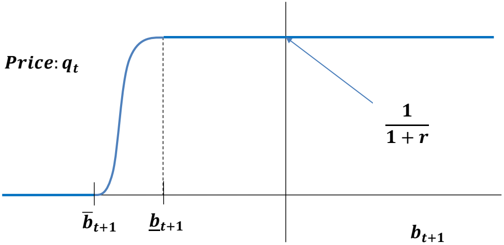

Sovereign's Problem

- Very similar

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Sovereign's Problem

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- FOC (relevant parts)

$$0 = -u'\left(y_t + b_t - q_t(b_{t+1})b_{t+1}\right) \times \left[q_t(b_{t+1}) + q_t'(b_{t+1})b_{t+1}\right] + \ldots$$

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- Two important terms: Quantity effect and price effect
  1. $q_t(b_{t+1})$: 1 more unit of debt $\implies$ q_t more consumption
  2. $q_t'(b_{t+1})b_{t+1}$: 1 more unit of debt $\implies$ Depress price for *whole* stock of debt by $q_t'(b_{t+1})$

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- Latter term is monopoly factor (*internalize* price changes)
  - Monopoly force: *Very important*
  - Determines how far 'over the cliff' he chooses to issue

Implications

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- Latter tends to dominate, especially when impatient
  - Borrowing in good times; saving in bad $\implies$ very volatile consumption process, countercyclical NX, etc.
  - All features of emerging market economies

Default Costs

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    - Countries tend to face worse consequences than autarky in default
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2. More debt sustained: Greater $\phi \implies$ lower $V_{d,t+1}$
   - Makes both $\bar{b}_{t+1}$ and $\underline{b}_{t+1}$ more negative

Role of Beliefs

- Common notion: Defaults sometimes caused by 'panics'
  - Fundamentals (i.e. technology, preferences) not responsible for default
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- Explore a couple of different ways
  - 'Laffer'-curve multiplicity
  - Liquidity crises

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  - High spreads *and* increased borrowing
  - Beliefs seemed to play a role e.g. third-party intervention successful
- High borrowing can be cause and consequence of beliefs and spreads

The 'Debt-Laffer' Curve

- Consider revenue from auctioning off b_{t+1}

$$Rev_t(b_{t+1}) = -q_t(b_{t+1})b_{t+1}$$

- Notice
  1. $Rev_t(0) = 0$
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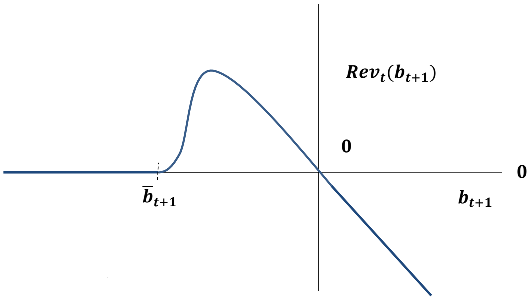
- In words, auction revenue is 'hump-shaped'
  - Much like tax-revenue as a function of tax rate (original Laffer curve)
- Intuition: Initially debt raises revenue, but
  - Issuance of a lot of debt lowers price $\implies$ Works to reduce revenue
  - Too much debt sends price all the way to zero

The 'Debt-Laffer' Curve



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- This model same as previous model if treasury always on 'good' side of Laffer curve i.e. b_L
- Sudden shift in expectations after consumption chosen could force b_H

Liquidity Crises

- Laffer curve not only way to generate belief-driven crises
- (Arguably) more common: Liquidity crises
- Akin to a bank run on the country
    - Lenders freeze up; refuse to invest
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  - Both feed off each other's behavior
- Change a couple of things to get these
  - Get rid of uncertainty (no m_{t+1})
  - Change timing
    1. Default decision takes place *after* debt auction
    2. Limited commitment in period t instead of $t+1$; can't commit to immediately run away with auction revenue

Finding Equilibria I

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- Notice
    - $\hat{V}_t(b_t)$ is equivalent to having lenders offer $\frac{1}{1+r}$ but setting $b_{t+1} = 0$
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- Debt thresholds
  - $V_t(\bar{b}_t) = V_{A,t}$
  - $\hat{V}_t(\underline{b}_t) = V_{A,t}$
- Since $\hat{V}_t(b_t) \leq V_t(b_t)$, it follows that

$$\bar{b}_t \leq \underline{b}_t$$

Characterizing Equilibria II

Three cases

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3. $b_t \in [\bar{b}_t, \underline{b}_t)$
   - $\implies \hat{V}_t(b_t) < V_{A,t} \leq V_t(b_t)$
   - Repayment depends on lender beliefs
   - Two equilibria
     3.1 Default if lenders expect default
     3.2 Repay if lenders expect repayment
   - This region often called ‘crisis zone’; always exists