FISCAL RULES AND LONG-TERM SOVEREIGN DEBT: THE CONSEQUENCES OF THE LIFETIME-LAFFER CURVE

ONLINE APPENDIX

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A Alternative Debt Contract Specifications

Here I provide variations on the key sufficiency results for the Lifetime-Laffer Curve to exhibit multiplicity for two alternative frameworks commonly used to model sovereign debt with a longer maturity. For simplicity, I will assume here as well that $\mathcal{S}_t = \emptyset$ for all t, though the results generalize to the case when this is not the empty set.

Were we to define our notion of Markov Perfect Equilibria in either of these environments all of the other results from the paper would go through as well.

A.1 Hatchondo and Martinez (2009)

The first specification is that of Hatchondo and Martinez (2009). In this framework, b is interpreted as the set of all current coupon obligations, which are assumed to decay at a rate δ. This set-up has the advantageous feature that it can be described by a single parameter, δ. We can write our two key equations in this context as

$$s(b_t) + q_t[b_{t+1} - (1-\delta)b_t] = b_t$$
$$q_t = \frac{[1 - g(b_{t+1})][1 + (1-\delta)q_{t+1}]}{1+r}$$

By performing the same derivations as in Section 2 of the paper, we arrive at a Lifetime-Laffer Curve described by

$$\frac{s(\bar{b})}{\bar{b}} + q(\bar{b})\delta = 1$$

where

$$q(\bar{b}) = \frac{1 - g(\bar{b})}{r + g(\bar{b}) + \delta[1 - g(\bar{b})]}$$

The alternative assumptions needed to ensure multiplicity are as follows:

Assumption A.1.1. $s(\bar{b})/\bar{b}$ *is a continuous and differentiable function for all* $\bar{b} \geq 0$.

Assumption A.1.2. $\lim_{\bar{b}\to 0} s(\bar{b})/\bar{b} \leq \frac{r}{r+\delta}$

Assumption A.1.3. $s(b_u)/b_u < 1$

As in the benchmark model, these assumptions imply that some debt issuance will be required in any solution, since average debt issuance will not be enough. With it, we can demonstrate the equivalent of Proposition 1. The feasibility condition here will be that there exists $\hat{b} \in (0, b_u)$ such that

$$\frac{s(\hat{b})}{\hat{b}} + q(\hat{b})\delta > 1$$

A variation on Proposition 2 holds as well.

A.2 Chatterjee and Eyigungor (2012)

Similar results can be derived for the framework of Chatterjee and Eyigungor (2012), in which the sovereign issues debt that matures stochastically at a rate λ and pays a coupon z in every period in which it does not mature. In this framework, we can write our two key equations as

$$s(b_t) + q_t[b_{t+1} - (1-\lambda)b_t] = [\lambda + (1-\lambda)z]b_t$$
$$q_t = \frac{[1 - g(b_{t+1})][\lambda + (1-\lambda)[z + q_{t+1}]]}{1+r}$$

By performing the same derivations as in Section 2, we arrive at a Lifetime-Laffer Curve described by

$$\frac{1}{\lambda+(1-\lambda)z}\frac{s(\bar{b})}{\bar{b}}+\frac{\lambda}{\lambda+(1-\lambda)z}q(\bar{b})=1$$

where

$$q(\bar{b})=\frac{[1-g(\bar{b})][\lambda+(1-\lambda)z]}{r+g(\bar{b})+\lambda[1-g(\bar{b})]}$$

The alternative assumptions needed to ensure multiplicity are as follows:

Assumption A.2.1. *$s(\bar{b})/\bar{b}$ is a continuous and differentiable function for all $\bar{b}\geq 0$.*

Assumption A.2.2. $\lim_{\bar{b}\to 0} s(\bar{b})/\bar{b}\leq \frac{r}{r+\lambda}\left[\lambda+(1-\lambda)z\right]$

Assumption A.2.3. $s(b_u)/b_u<\lambda+(1-\lambda)z$

As in the benchmark model, these assumptions imply that some debt issuance will be required in any solution, since average debt issuance will not be enough. With it, we can demonstrate the equivalent of Proposition 1. The feasibility condition here will be that there exists $\hat{b}\in(0,b_u)$ such that

$$\frac{1}{\lambda+(1-\lambda)z}\frac{s(\hat{b})}{\hat{b}}+\frac{\lambda}{\lambda+(1-\lambda)z}q(\hat{b})>1$$

A variation on Proposition 2 holds as well.

B Commitment and the Approximating Model

Recall the approximating model.

$$\frac{s(b_t)}{\Delta} \; + \; q_t \times \left[b_{t+1/\Delta} - \left(1 - \frac{\lambda}{\Delta}\right) b_t \right] = \frac{\lambda}{\Delta} b_t \tag{C.1}$$

$$q_t = \frac{1}{1 + \frac{r}{\Delta}} \times \Bigg[\left[1 - g(b_{t+1/\Delta})\right] \times \left(\frac{\lambda}{\Delta} + \left(1 - \frac{\lambda}{\Delta}\right) q_{t+1/\Delta} \right) + \tag{C.2}$$
$$g(b_{t+1/\Delta}) \times \left(0 + \left(1 - \frac{1}{\Delta}\right) q_{t+1/\Delta} \right) \Bigg]$$

This approximating model essentially splits the original model into subperiods of length $1/\Delta$. Importantly, the Lifetime-Laffer Curve is preserved exactly i.e. if one were to compute the set of steady states in this model, they would be exactly the same as in the original model outlined in Section 2. In fact, that model is simply a special case of this one when $\Delta = 1$.

The equations governing the behavior of the approximating problem create a relatively clear picture of the environment. A subperiod will have length $1/\Delta$. During this subperiod, the surplus will be the standard prescription for the period-wide primary surplus at that debt level divided by the number of subperiods. Thus, if the sovereign maintained a constant debt level for Δ contiguous subperiods, the final surplus in a period would be the same in both models. The same is true for maturing debt i.e. in the general model a fraction λ of the debt matures in a period and in the approximating model a fraction λ/Δ matures in a given subperiod.

With regard to the default decision, the game is played slightly differently in the approximating model. The sovereign can default in any subperiod, and does so according to the same function $g(\cdot)$. Should he choose not to default, he pays the fraction of the maturing principal, λ/Δ. An individual creditor thus receives λ/Δ per unit of debt and enters the next period with $1 - \lambda/\Delta$ outstanding claims on the sovereign. Should the sovereign choose to default, however, two things occur. First, the creditors receive no payment in that period. Second, the creditors' debt claim is slashed by a fraction $1/\Delta$. There is no period of exclusion and the game resumes in the next subperiod. Importantly, their claim on the sovereign is not obliterated completely when $\Delta > 1$, as it is in the general model when $\Delta = 1$.

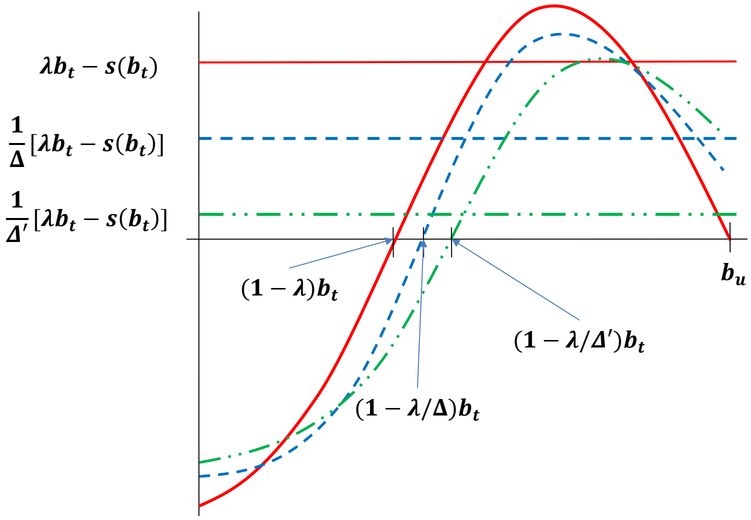
This minimum guaranteed return is what makes the approximating model a useful approximation since it will serve to remove solutions on the inferior side of the contemporaneous Laffer curve. The inferior side of the Laffer curve falls back to zero since lenders eventually anticipate that their claim on the debt will be completely eradicated; since this is not true in the approximating model, any Pareto-inferior solution will eventually disappear. In fact as Δ goes up, two things happen: The right-hand-side of the Laffer curve lifts above the x-axis and revenue needs fall closer to the x-axis. This can be seen in Figure 1. These two forces work in tandem to eliminate inferior solutions and guarantee commitment in the approximating model.

B.1 Applicability

This approximating problem exhibits similar, though not identical, stability properties to the general model. However, for low enough levels of Δ, we know that they will be the same.

Result B.1. *Suppose that the characteristic roots of the general model are positive and real at all steady states and that there are a finite number of steady states. There exists a $\Delta' > 1$ such that for $\Delta \in [1, \Delta']$, the characteristic roots of the approximating model imply the same stability properties as the general model.*

Figure 1:
The Logic of the Approximating Model



Proof. The characteristic roots of a candidate solution to the Lifetime-Laffer Curve, $(\bar{q}, \bar{b})$, are given by:

$$\hat{\lambda}_1 = \frac{B + \sqrt{B^2 - 4D}}{2}$$
$$\hat{\lambda}_2 = \frac{B - \sqrt{B^2 - 4D}}{2}$$

where

$$B = \left(1 - \frac{\lambda}{\Delta}\right) + \frac{\lambda - s'(\bar{b})}{\bar{q}\Delta} + \frac{\left(1 + \frac{r}{\Delta}\right)}{(1 - \lambda/\Delta) - g(\bar{b})(1 - \lambda)/\Delta}$$

$$- \frac{1}{\bar{q}\Delta}\lambda\bar{b} \times \left(\frac{g'(\bar{b})\frac{\lambda}{\Delta}}{(1 - \lambda/\Delta) - g(\bar{b})(1 - \lambda)/\Delta} + \frac{\left[\bar{q}\left(1 + \frac{r}{\Delta}\right) - [1 - g(\bar{b})]\frac{\lambda}{\Delta}\right] g'(\bar{b})\frac{1-\lambda}{\Delta}}{\left[(1 - \lambda/\Delta) - g(\bar{b})(1 - \lambda)/\Delta\right]^2} \right)$$

$$D = \left(\left(1 - \frac{\lambda}{\Delta}\right) + \frac{\lambda - s'(\bar{b})}{\bar{q}\Delta}\right) \times \left(\frac{\left(1 + \frac{r}{\Delta}\right)}{(1 - \lambda/\Delta) - g(\bar{b})(1 - \lambda)/\Delta}\right)$$

To see this, we first observe that $\lim_{\Delta \to \infty} B = 2$ and $\lim_{\Delta \to \infty} D = 1$. This implies that $\lim_{\Delta \to \infty} \hat{\lambda}_{i,s} = 1$ for $i = 1, 2$ and for any steady state, s. Notice further that both B and D and, by extension, $\hat{\lambda}_{i,s}$, are continuous in Δ. Let $\hat{\lambda}_{i,s}(\Delta)$ denote a characteristic root as a function of Δ.

Suppose first that some root has the feature $\hat{\lambda}_{i,s}(1) < 1$. We know that $\lim_{\Delta \to \infty} \hat{\lambda}_{i,s} = 1$. We cannot say whether or not the root approaches unity in a monotone fashion, but by continuity, there must be some $\Delta_{i,s} \in (1, \infty)$ such that $\hat{\lambda}_{i,s}(1) < 1$. Further, it must be the case that at least one such $\Delta_{i,s}$ exists such that $\forall \Delta \in [1, \Delta_{i,s}]$, we have that $\hat{\lambda}_{i,s}(\Delta) < 1$. A symmetric argument can be made whenever $\hat{\lambda}_{i,s}(1) > 1$.

Thus, we can choose $\Delta' = \min_{i,s} \Delta_{i,s}$. Since the number of steady states is finite, Δ' exists and is strictly greater than one. □

References

[1] **Chatterjee, Satyajit and Burcu Eyigungor**, "Maturity, Indebtedness, and Default Risk," *American Economic Review*, 2012, *102* (6), 2674–2699.

[2] **Hatchondo, Juan Carlos and Leonardo Martinez**, "Long-Duration Bonds and Sovereign Default," *Journal of International Economics*, 2009, *79* (1), 117–125.